Assignment 4 - Question 3

(from KT chap 7) In the exciting beginning of the internet, many people would say that much of the large potential in a company like AOL was in the “eyeballs” - the simple fact that millions of people look at its pages every day. Further, by convincing people to register personal data with the site, a site like AOL can show each use an extremely targeted advertisement whenever he or she visits the site, in a way that TV networks or magazines couldn’t hope to match. So if a user has told AOL that he or she is a 20-year-old computer science major from Cornell University, the site can present a banner as for apartments in Ithaca, NY; on the other hand, if she or he is a 50-year-old investment banker from Greenwich, CT, the site can display a banner ad pitching Lincoln Town Cars instead.

But deciding on which ads to show to which people involves some serious computation behind the scenes. Suppose that the managers of a popular web site have identified $k$ distinct demographic groups $G_1, G_2, \ldots, G_k$. (These groups can overlap; for example, $G_1$ can be equal to all residents of New York State and $G_2$ can be equal to all people with a degree in computer science.) The site has contracts with $m$ different advertisers to show a certain number of copies of their ads to users of the site. Here’s what the contract with the $i^{th}$ advertiser looks like:

- For a subset $X_i \subseteq \{G_1, G_2, \ldots, G_k\}$ of the demographic groups, advertiser $i$ wants its ads shown only to users who belong to at least one of the demographic groups in $X_i$.
- For a number $r_i$, advertiser $i$ wants its ads shown to at least $r_i$ users each minute.

Now consider the problem of designing a good advertising policy - a way to show a single ad to each user of the site. Suppose at a given minute there are $n$ users visiting the site. Because we have registration information on each of these users, we know that user $j$ (for $j = 1, 2, \ldots, n$) belongs to a subset $U_j \subseteq \{G_1, G_2, \ldots, G_k\}$ of the demographic groups. The problem is: Is there a way to show a single ad to each user so that the site’s contracts with each of the $m$ advertisers is satisfied for this minute? (That is, for each $i = 1, 2, \ldots, m$ can at least $r_i$ of the $n$ users, each belonging to at least one demographic group in $X_i$, be shown an ad provided by advertiser $i$?)

Give an efficient algorithm to decide if this is possible, and if so, to actually choose an ad to show each user.