(from Er) Suppose you are given a collection of up-trees representing a partition of the set \{1, 2, \ldots, n\} into disjoint subsets. **You have no idea how these trees were constructed.** You are also given an array `node[1 \ldots n]`, where `node[i]` is a pointer to the up-tree node containing element `i`. Your task is to create a new array `label[1 \ldots n]` using the following algorithm `LABELEVERYTHING`:

```plaintext
for i=1 to n
    label[i] = Find(node[i])
```

Prove that if we implement `FIND` using path compression, then all `n` operations are executed in \(O(n)\) time in total.