Assignment 1

due April 11, 2022

1. For this problem imagine that you work for a lab which is studying the famous gastropods (snails and slugs) of Oregon. They have a sample of \( n \) gastropods, \( g_1, g_2, \ldots, g_n \), and the researchers working there have made a series of \( s \) slimy determinations that two given gastropods belong to different species. Here a determination is of the form \((i, j)\), meaning that \( g_i \) and \( g_j \) belong to different species. Your job is to give an \( O(n + s) \) time algorithm to decide whether these determinations are consistent with the gastropods belonging to just two species. Notes: (i) It is possible that they could belong to three or more species, but that is a different question. (ii) You will need to build graph since the input is a list. [8 points]

2. Here modify your solution to the previous problem to make the same test of whether the gastropods could belong to just two species, but where the types of determinations tell you **SAME** or **DIFF**. That is, you are given a list of \( s \) determinations of the form

- \((i, j, \text{DIFF})\) indicating that \( g_i \) and \( g_j \) belong to different species, or
- \((i, j, \text{SAME})\) indicating \( g_i \) and \( g_j \) belong to the same species

Note that not every pair \( g_i \) and \( g_j \) have a determination, for some the researchers found that a comparison could go either way. [7 points]

3. The California Gold Rush (1848-1855) was one of the largest mass migrations in human history. Forty-niners came from across the globe – Oregon, China, Australia, Latin America, Hawaii, Europe, etc. – seeking to strike it rich mining for gold in the Sierra Nevada mountains. For a combination of reasons (illiteracy, language difference, secrecy, loss), there is no written record of the years each mine has operated. As such, a group of local historians is trying to infer when each mine operated from alternate records. As a starting point, these historians are focusing on a specific set of \( n \) mines, and by cross-referencing various sources, these historians collected an initial set of records. Each record takes one of two possible forms, namely:

(a) That two mines from this set of \( n \) operated simultaneously.
(b) The one mine from the set of \( n \) closed before another mine (from the set of \( n \)) opened.

Assume that once a mine stopped operating, it remained closed forever.

Before considering additional mines beyond the \( n \), they hired you to check whether their records have any internal contradictions. Provide an efficient algorithm that determines whether their records contain such a contradiction. Provide your algorithm’s runtime. 

*Hint*: Map the records to a directed graph, and represent each mine as two connected nodes. [10 points]

Total: 25 points