1. Let HamPath be the following problem: Given an undirected graph $G$, does $G$ have a Hamilton path? Recall that a Hamilton path is one the visits every node exactly once. The HamCycle problem has the same input but is asking whether the graph has a Hamilton cycle.

Show that $\text{HamPath} \equiv_p \text{HamCycle}$.

Note that this is really two questions:

- $\text{HamPath} \leq_p \text{HamCycle}$
- $\text{HamCycle} \leq_p \text{HamPath}$

2. We will write the reduction $\leq_P$ defined in the text on p 453 as $\leq^P_T$, denoting a polynomial-time Turing reduction. (This distinguishes it from $\leq^P_m$ defined in class, used implicitly in the text.)

Suppose we have a fixed alphabet $\Sigma = \{0, 1\}$. For a set $S \subseteq \Sigma^*$, denote the complement of $S$ by $\bar{S} = \Sigma^* - S$. Suppose that

- $A, B \subseteq \Sigma^*$
- $B \in \text{NP}$, $\bar{B} \in \text{NP}$
- $A \leq^P_T B$

Prove that $A \in \text{NP}$ and $\bar{A} \in \text{NP}$. 