Assignment 4

due Wednesday, February 17, 2021

For the problems below, just give a description of the subproblem and a recurrence relation for the optimal solution value. There is no need to write any code (and none is desired).

1. chapter 6, exercise 6, pp 317-318

2. chapter 6, exercise 19, p 329

3. (look at exercise 6.26, p 197, from DPV) Consider the following version of the sequence alignment problem. We have strings $X, Y \in \Sigma^*$, and the closeness of the alignment between $X = X_1X_2\ldots X_n$ and $Y = Y_1Y_2\ldots Y_m$ is determined by a scoring matrix $\delta$ of size $(|\Sigma| + 1) \times (|\Sigma| + 1)$, where the extra rows and columns are to accommodate gaps. We define a subproblem $AS(i,j)$ (“alignment score”) as the score of the highest-scoring alignment of $X = X_1X_2\ldots X_i$ and $Y = Y_1Y_2\ldots Y_j$.

   (a) Give a recurrence for $AS(i,j)$.

   (b) What values assigned to $\delta$ describe the LONGEST COMMON SUBSEQUENCE problem?

   (c) What values assigned to $\delta$ describe the EDIT DISTANCE problem (from the Dasgupta et al text, section 6.3, p 174)? (Hint: use some negative numbers.)

4. (exercise 6.6, DPV) We define a multiplication operation table on three symbols $a, b, c$ according to something like the table below, so that $ab = b$, $ca = a$, and so on. The goal is to determine whether a string of symbols, $s_1s_2\ldots s_n$ ($s_i \in \{a, b, c\}$) can be parenthesized in such a way so that their multiplication together equals $a$. Note that the operation is neither associative nor commutative (that is, it is possible that $ab \neq ba$ and $(ab)c \neq a(bc)$).

   Table 1: multiplication table for $a, b, c$

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>a</td>
<td>c</td>
<td>c</td>
</tr>
</tbody>
</table>

   For example, if the input string is $bbbab$ then the answer should be true since $((b(b)b))(ba)c = a$. Hint: define a subproblem $P[i, j, t]$, where $1 \leq i \leq j \leq n$ and $t \in \{a, b, c\}$. $P[i, j, t]$ will be true if it is possible to parenthesize $s_is_{i+1}\ldots s_j$ in such a way that the multiplication in that order equals the symbol $t$, and false otherwise.

5. The “Bone’s Battery” problem, linked to from class page. Remember - just subproblem, recurrence, no code.
Note: take a look at the following problem (from DPV). This will form the basis of the programming assignment - a sample recurrence will be provided.

(exercise 6.23, p 196, from DPV) A mission-critical production system has $n$ stages that have to be performed sequentially; stage $i$ is performed by machine $M_i$. Each machine $M_i$ has a probability $r_i$ of functioning reliably and a probability $1 - r_i$ of failing (and the failures are independent). Therefore, if we implement each stage with a single machine, the probability that the whole system works is $r_1 \cdot r_2 \cdots r_n$. To improve this probability we add redundancy by having $m_i$ copies of the machine $M_i$ that performs stage $i$. The probability that all $m_i$ copies fail simultaneously is only $(1 - r_i)^{m_i}$, so the probability that stage $i$ is completed correctly is $1 - (1 - r_i)^{m_i}$ and the probability that the whole system works is $\prod_{i=1}^{n} [1 - (1 - r_i)^{m_i}]$. Each machine $M_i$ has a cost $c_i$, and there is a total budget $B$ to buy machines. (Assume that $B$ and the $c_i$ are positive integers.)

Given the probabilities $r_1, r_2, \ldots, r_n$, the costs $c_1, c_2, \ldots, c_n$, and the budget $B$, find the maximum reliability that can be achieved within budget $B$. 