from Er) Suppose you are given a collection of up-trees representing a partition of the set \{1, 2, \ldots, n\} into disjoint subsets. You have no idea how these trees were constructed. You are also given an array \texttt{node}[1\ldots n], where \texttt{node}[i] is a pointer to the up-tree node containing element \texttt{i}. You task is to create a new array \texttt{label}[1\ldots n] using the following algorithm \texttt{LABEL\_EVERYTHING}:

\begin{verbatim}
for i=1 to n
    label[i] = Find(node[i])
\end{verbatim}

Prove that if we implement \texttt{FIND} using path compression, then all \texttt{n} operations are executed in \texttt{O(n)} time.