CIS 313, Intermediate Data Structures
Winter 2021

Assignment 1
due Tuesday, Jan 26, 2021

1. Suppose that algorithm $A$ uses $313 \cdot n^3$ operations while algorithm $B$ uses $2 \cdot n^4$ operations. Determine the smallest value $N$ such that $A$ is as fast or faster than $B$ for all $n \geq N$. [4 points]

2. exercise 3.1-2, p 52. Show that for any real constants $a$ and $b$, where $b > 0$: $(n+a)^b = \Theta(n^b)$. [6 points]

3. exercise 3-2, p 61.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
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</thead>
<tbody>
<tr>
<td>a. $\lg^c n$</td>
<td>$n^c$</td>
<td>?</td>
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<tr>
<td>b. $n^c$</td>
<td>$c^d$</td>
<td>?</td>
<td>?</td>
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<tr>
<td>c. $\sqrt{n}$</td>
<td>$n^{\sin{n}}$</td>
<td>?</td>
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<td>?</td>
<td>?</td>
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<tr>
<td>d. $2^n$</td>
<td>$2^{n/2}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
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<td>?</td>
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<tr>
<td>e. $n^{lg c}$</td>
<td>$c^{lg n}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
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<td>?</td>
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<tr>
<td>f. $\lg(n!)$</td>
<td>$\lg(n^n)$</td>
<td>?</td>
<td>?</td>
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[8 points]

4. An algorithm takes $0.4ms$ for input size 50 (this allows you to determine the constant $c$, which will be different in each case). What is the largest size of an input that can be solved in one hour if the run time of the algorithm is . . . ?

(a) $c \cdot n$
(b) $c \cdot n \log n$ (assuming base 2)
(c) $c \cdot n^3$
(d) $c \cdot 2^n$

[8 points]

5. exercise 2-3, p 41. [8 points]

6. Implement a stack using a single queue. In particular, you are given a queue $Q$ that provides the method $Q.size()$ to return its size at any point and the standard methods of queues (i.e, $Q.enqueue(x)$ and $Q.dequeue()$). The requirement is to use such methods of $Q$ to implement two methods $S.push(x)$ and $S.pop()$ for a stack $S$. What are the running times of your methods? [6 points]

Total: 40 points

Notes:
- *Hint*: Question 2: You need to show $(n + a)^b = O(n^b)$ and $(n + a)^b = \Omega(n^b)$. In both cases, you can choose $N = 2|a|$.

- *Hint*: Question 4 b: You don’t have to show the exact value. An interval to bound the value is enough. Try different values to find the interval.

- An *ms* is 1/1000 of a second, also called a millisecond.