Assignment 3

due Wednesday, February 12, 2020

1. Given \( n \) pairs \((x_0, y_0), (x_1, y_1), \ldots (x_{n-1}, y_{n-1})\) we want to find a polynomial \( A(x) \) such that, for all \( i \), \( A(x_i) = y_i \). As mentioned in class, Lagrange’s formula gives us a way to determine \( A \):

\[
A(x) = \sum_{k=0}^{n-1} y_k \frac{\prod_{j\neq k}(x-x_j)}{\prod_{j\neq k}(x_k-x_j)}.
\]

(a) Argue that, for any \( i \) \( (0 \leq i < n) \), \( A(x_i) = y_i \).

(b) Show how to compute the coefficient representation of \( A(x) \) in \( O(n^2) \) time.

2. chapter 5, exercise 3, pp 246-247

3. chapter 5, exercise 7, pp 248-249

4. chapter 6, exercise 2(b), pp 313-314

Comments:

- The question for problem 1 is not to be able to evaluate \( A \) on an \( x \), but to determine the coefficients of \( A \). View the problem as taking input \((x_0, y_0), (x_1, y_1), \ldots (x_{n-1}, y_{n-1})\) and returning as output \( a_n, a_{n-1}, \ldots, a_1, a_0 \).

- Also on problem 1, a base operation is integer multiplication and/or addition. That is, if \( x \) and \( y \) are integers, then \( x \cdot y \) and \( x + y \) are \( O(1) \). On the other hand, if \( p(x) \) is a polynomial of degree \( n \), presented by its \( n + 1 \) coefficients, and \( q(x) \) is a polynomial of degree \( m \), then computing \( r(x) = p(x) \cdot q(x) \) (that is, determine the \( n + m + 1 \) coefficients of \( r(x) \)) will take time \( O(n \cdot m) \) by the usual method.

- Finally for problem 1, if you look online you may see a reference to barycentric form. Don’t go there. Just explain in your own words and/or pseudo code.

- For question 4, you do not need to do part (a), but you should read it.