1. (exercise 17.4-3, CLRS, p 471, reworded)

Suppose that for the Dynamic Table problem allowing Insert and Delete, we
- double the size of the table when it is full
- contract the table by multiplying its size by $\left(\frac{2}{3}\right)^k$ when its load factor drops below $\frac{1}{3}$
- where $num$ indicates the number of elements stored in the table and $size$ the number of allocated slots (both full and empty), use the potential function $\Phi = |2 \cdot num - size|$ (the notation $|x|$ indicates the absolute value of $x$).

Derive amortized cost bounds for the Insert and Delete operations. The cost basis for the array expansion is just the copy operations, no charge for initializing anything to zero.

(Note) Using the $f$ (full) and $e$ (empty) measures from class, we can rewrite the potential by substituting $num = f$ and $size = f + e$. Now $\Phi = |f - e|$.

2. (exercise 19, part (a) from chap 11 of Er, reworded)

You are given an undirected graph $G = (V, E)$ with start and finish nodes $s, t \in V$. The nodes represent locations and edges are roads between the locations along which people can walk. Each edge/road of $G$ is gated, allowing at most one person to walk through per hour: for example one person can follow that edge between 1:00 and 1:05, and then another between 2:00 and 2:05.

Given an integer $h$, describe an algorithm and give the time to determine the maximum number of people who can walk from $s$ to $t$ in $h$ hours. (The hint from Er indicates that you need to build a new graph and the time bound will include $h$.)

3. Consider a directed flow graph $G = (V, E)$ with capacity function $c : E \to \mathbb{N}$. Let $G^r$ be the reversal of $G$:
- $G^r = (V, E^r)$ where $E^r = \{(u \to v) \mid (v \to u) \in E\}$
- the capacity function of $G^r$ is $c^r(u \to v) = c(v \to u)$

Is the max-flow in $G$ the same as the max-flow in $G^r$ when

(a) $G$ is a DAG (acyclic)?
(b) $G$ is a general directed graph, allowing cycles?

4. Suppose we have three sets $S_0, S_1, S_3 \subseteq U$ represented by Bloom filters $B_0, B_1, B_3$ respectively. All sets have $n$ elements and all filters are of size $m$ and were constructed using the same set of hash functions $h_1, h_2, \ldots, h_k : U \to [m]$. (The parameters $n, m, k$ are the same as used in the Er text.)
(a) Let \( B \) be the bit-wise AND of the vectors \( B_0 \) and \( B_1 \), which represents the set \( S = S_0 \cap S_1 \). What is the false positive rate for a query to \( B \)?

(b) Let \( B \) be the bit-wise OR of the vectors \( B_0 \) and \( B_1 \), which represents the set \( S = S_0 \cup S_1 \). What is the false positive rate for a query to \( B \)?

(c) Suppose that you manage to construct a set \( S \subseteq \mathcal{U} \) of size \( n \) such that by storing it in a Bloom filter \( B \) using the hash functions above you obtain \( B = B_3 \) (that is, the vectors have the same pattern of zeroes and ones). You want to say you have a "copy" of \( S_3 \). What is the probability that \( S = S_3 \)?