1. (exercise 14 from chap 11 of Er)
   You’re organizing the First Annual CSGSBS* Dance, to be held all day Friday, Saturday, and Sunday. Several 30-minute sets of music will be played during the event, and a large number of DJs have applied to perform. You need to hire DJs according to the following constraints:
   - Exactly $k$ sets of music must be played each day, and thus $3k$ sets altogether.
   - Each set must be played by a single DJ in a consistent music genre (ambient, bubblegum, dubstep, horrorcore, K-pop, etc.)
   - Each genre must be played at most once per day.
   - Each candidate DJ has given you a list of genres they are willing to play.
   - Each DJ can play at most three sets during the entire event.

   Suppose there are $n$ candidate DJs and $g$ different musical genres available. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the $3k$ sets, or correctly reports that no such assignment is possible.

   (*) Computer Science Graduate Student Benevolent Society

2. (exercise 18 from chap 11 of Er)
   Faced with budget cuts, Potemkin University (PU), has decided to hire actors to sit in on classes as “students” to ensure that every class they offer is completely full. Because actors are expensive, the university wants to hire as few of them as possible. So here, the administrators at PU have given you a directed acyclic graph $G = (V, E)$ whose vertices represent classes and where each edge $i \rightarrow j$ indicates that the same “student” can attend class $i$ and then later attend class $j$. In addition, you are also given an array $cap[1 \ldots V]$ listing the maximum number of “students” who can take each class. Describe and analyze an algorithm to compute the minimum number of “students” that would allow every class to be filled to capacity.

3. (exercise 26-2, CLRS, p 761)
   A path cover of a directed graph $G = (V, E)$ is a set $P$ of vertex-disjoint paths such that every vertex in $V$ is included in exactly one path in $P$. Paths may start and end anywhere, and they may be of any length, including 0. A minimum path cover of $G$ is a path cover containing the fewest possible paths.

   (a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$. (Hint: Assuming that $V = \{1, 2, \ldots, n\}$, construct the graph $G' = (V', E')$, where $V' = \{x_0, x_1, \ldots, x_n\} \cup \{y_0, y_1, \ldots, y_n\}$, $E' = \{(x_i, x_j)|i \in V\} \cup \{(y_i, y_j)|i \in V\} \cup \{(x_i, y_j)|(i, j) \in V\}$, and run a maximum-flow algorithm.)

   (b) Does your algorithm work for directed graphs that contain cycles? Explain.