1. (question 4 from chap 10 of Er) Let $G$ be a flow network that contains an opposing pair of edges $u \to v$ and $v \to u$, both with positive capacity. Let $G'$ be the flow network obtained from $G$ by decreasing the capacities of both these edges by $\min\{c(u \to v), c(v \to u)\}$. In other words:

- if $c(u \to v) > c(v \to u)$, change the capacity of $u \to v$ to $c(u \to v) - c(v \to u)$ and delete $v \to u$.
- if $c(u \to v) < c(v \to u)$, change the capacity of $v \to u$ to $c(v \to u) - c(u \to v)$ and delete $u \to v$.
- if $c(u \to v) = c(v \to u)$, delete both $u \to v$ and $v \to u$.

(a) Prove that every maximum $(s, t)$-flow in $G'$ is also a maximum $(s, t)$-flow in $G$. (Thus, by simplifying every opposing pair of edges in $G$, we obtain a new reduced flow network with the same maximum flow value as $G$.)

(b) Prove that every minimum $(s, t)$-cut in $G$ is also a minimum $(s, t)$-cut in $G'$ and vice versa.

(c) (for 513) Prove that there is at least one maximum $(s, t)$-flow in $G$ that is not a maximum $(s, t)$-flow in $G'$. 
