1. Suppose you are given the adjacency matrix representation $M$ of a directed graph $G = (V, E)$. Note that the size of $M$ is $\Theta(n^2)$. The goal here is to determine if there is a node of $G$ with in-degree $n - 1$ and out-degree 0 (that is, all other nodes point to it and it points to no other node). Give an algorithm to do this which runs in $\Theta(n)$ time (so not $\Theta(n^2)$). [5 points]

2. Suppose you work for a lab which is studying butterflies. It has a sample of $n$ butterflies, $L_1, L_2, \ldots, L_n$. The researchers have made a series of $r$ determinations determining whether two butterflies belong to different species. A determination is of the form $(i, j)$, and it means that $L_i$ and $L_j$ belong to different species. Your job is to give an $O(n + r)$ time algorithm to decide whether the determinations are consistent with the butterflies belonging to just two species. (Note: it is possible that they could belong to three or more species, but that is a separate question.) [5 points]

3. exercise 22.3-2, from CLRS text [5 points]

4. exercise 22.4-1, from CLRS text [5 points]

5. exercise 3-16, from DPV text [5 points]

Total: 25 points