\[ f(n) = \frac{\log (n^2 - \log n)}{\log n} + \frac{\log ^2 \left( \frac{n}{2} + \sqrt{n} \right)}{\log ^2 (n)} \]

\[ = \Theta \left( \log n \right) \]
\( n > 0 \)  
out: \( n \frac{(n+1)}{2} \)

- **Init**
  - \( S = 0 \)
  - \( k = 0 \)
- **Loop**
  - While \( k \leq n + 1 \)
  - \( S = S + k \)
  - \( k = k + 1 \)
- **End**
  - returns \( S \)

\[ f: k \leq n + 1 \]

\[ a: 0 \leq k \leq n + 1 \]

\[ S = \frac{k(k-1)}{2} \]

**Correctness Proof:**

1. **Init**: Want: \( k \) is correct at the beginning.
   - We have:
     - At the beginning: \( S = 0 \), \( k = 0 \)
   - \( 0 \leq k \leq n + 1 \) \( \checkmark \)
   - \( S = \frac{k(k-1)}{2} \) \( \checkmark \) \( \text{but } S = k = 0. \)

2. **Maintain in one step**:
   - Want: If \( \alpha \) and \( \gamma \) are correct
     - then \( \delta \) is still correct after one iteration of the While loop.

\[ \text{ble } \alpha \text{ and } \gamma \text{ are correct } < k \leq n + 1 (\beta) \]

after one step of While, call \( S, k' \): new values of \( S \) and \( k' \).
Due to

While loop:

need to show:

\[
\frac{0 \leq k' \leq n+1}{\iff}
\]

\[
S' = \frac{k'(k'-1)}{2}
\]

\[
S = \frac{k(k+1)}{2}
\]

\[
k' = k+1 > 0
\]

\[
k' = k+1 < n+2 \rightarrow k' \leq n+1
\]

\[
S' = S + k = \frac{k(k+1)}{2} + k = \frac{k^2}{2} + k = \frac{k^2 - k + 2k}{2}
\]


3. Termination step:

- Show the loop will terminate

- \[
T \Rightarrow x \rightarrow S = \frac{n(n+1)}{2}
\]

From \( T \): we have: \( k > n+1 \Rightarrow k = n+1 \)

From \( x \): \( 0 \leq k \leq n+1 \)

\[
S = \frac{(n+1)(n+1-1)}{2} = \frac{n(n+1)}{2}
\]

Done!
\[ n = b[0].2^0 + b[1].2^1 + \cdots + b[k].2^k \]