Assignment 1

due Tuesday, Jan 28, 2020

1. Suppose that algorithm $A$ uses $313 \cdot n^3$ operations while algorithm $B$ uses $2 \cdot n^4$ operations. Determine the smallest value $N$ such that $A$ is as fast or faster than $B$ for all $n \geq N$. [4 points]

2. exercise 3.1-2, p 52. Show that for any real constants $a$ and $b$, where $b > 0$: $(n + a)^b = \Theta(n^b)$. [6 points]

3. exercise 3-2, p 61.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>O</th>
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<th>Ω</th>
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</thead>
<tbody>
<tr>
<td>a. $\log^k n$</td>
<td>$n^k$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>b. $n^k$</td>
<td>$c^k$</td>
<td>?</td>
<td>?</td>
<td>?</td>
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<td>?</td>
</tr>
<tr>
<td>c. $\sqrt{n}$</td>
<td>$n^{\sin n}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>d. $2^n$</td>
<td>$2^{n/2}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
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<tr>
<td>e. $n^{\lg c}$</td>
<td>$c^{\lg n}$</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
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<tr>
<td>f. $\lg(n!)$</td>
<td>$\lg(n^n)$</td>
<td>?</td>
<td>?</td>
<td>?</td>
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   [8 points]

4. An algorithm takes $0.4ms$ for input size 50 (this allows you to determine the constant $c$, which will be different in each case). What is the largest size of an input that can be solved in one hour if the run time of the algorithm is . . . ?

   (a) $c \cdot n$
   (b) $c \cdot n \log n$ (assuming base 2)
   (c) $c \cdot n^3$
   (d) $c \cdot 2^n$

   [8 points]

5. exercise 2-3, p 41. [8 points]

6. Implement a stack using a single queue. In particular, you are given a queue $Q$ that provides the method $Q.size()$ to return its size at any point and the standard methods of queues (i.e, $Q.enqueue(x)$ and $Q.dequeue()$). The requirement is to use such methods of $Q$ to implement two methods $S.push(x)$ and $S.pop()$ for a stack $S$. What are the running times of your methods? [6 points]

Total: 40 points

Notes:
• \textit{Hint}: Question 2: You need to show \((n + a)^b = O(n^b)\) and \((n + a)^b = \Omega(n^b)\). In both cases, you can choose \(N = 2|a|\).

• \textit{Hint}: Question 4 b: You don’t have to show the exact value. An interval to bound the value is enough. Try different values to find the interval.

• An \textit{ms} is 1/1000 of a second, also called a millisecond.