Written Assignment 3

Deadline: May 17, 2020

**Instruction:** You may discuss these problems with classmates, but please complete the write-ups individually. Your answers must be **typewritten**, except for figures, which may be hand-drawn. Please submit your answers (pdf format only for non-programming assignments) on Canvas.

**Q1. Coalitional Games (50 points)**

**Shapley and Core (20pts).** Consider the following coalitional game with 3 players.

\[
\begin{align*}
  v(\{1\}) &= 5, \quad v(\{2\}) = 10, \quad v(\{3\}) = 15 \\
  v(\{1,2\}) &= 20, \quad v(\{1,3\}) = 20, \quad v(\{2,3\}) = 30 \\
  v(\{1,2,3\}) &= 40
\end{align*}
\]

- Provide the Shapley values for each player.
- Provide a stable outcome of the payoff distribution. If no stable outcome exists, write None.

**Optimal Coalitional Structure (30pts).** Consider the following coalitional game with 4 players.

\[
\begin{align*}
  v(\{1\}) &= 40, \quad v(\{2\}) = 25, \quad v(\{3\}) = 45, \quad v(\{4\}) = 35 \\
  v(\{1,2\}) &= 55, \quad v(\{1,3\}) = 70, \quad v(\{1,4\}) = 50, \quad v(\{2,3\}) = 80, \quad v(\{2,4\}) = 65, \quad v(\{3,4\}) = 85 \\
  v(\{1,2,3\}) &= 115, \quad v(\{1,2,4\}) = 85, \quad v(\{1,3,4\}) = 130, \quad v(\{2,3,4\}) = 100 \\
  v(\{1,2,3,4\}) &= 160
\end{align*}
\]

Provide the optimal coalitional structure of the game.

**Q2. Auction Games (35 points)**

Suppose a seller runs a second-price, sealed-bid auction for a painting. There are two bidders with independent, private values. The seller does not know their precise valuations, but knows: (a) each bidder \(i\) has one of three values, \(v_i = 2, v_i = 4\) or \(v_i = 8\); and (b) each of these values is equally likely (i.e., occurs with probability \(\frac{1}{3}\)). When running the auction, if the two bids are tied (say, at \(x\)), the winner is chosen at random (and pays \(x\)).

1. (10pts) Assume both bidders use their dominant strategies for bidding in a second-price auction. What is the seller’s expected revenue in this auction? Please explain your answer.
2. (20pts) Now the seller decides to set a reserve price of \( r \). This means that if the highest bid is below \( r \), the seller will not sell the item. If the highest bid is at least \( r \), then the painting will go to the highest bidder, and the winner will pay the maximum of \( r \) and the second-highest bid. Suppose the reserve price is set to \( r = 4 \). Assume both bidders use their dominant strategies. What is the seller’s expected revenue in this auction? Please explain your answer. If the expected revenue increases or decreases relative to your answer in part (1), give a qualitative explanation for why this change occurs.

3. (5pts) Is there a better reserve price than \( r = 4 \) (i.e., that will provide more revenue for the seller)? Give a brief justification for your response.

Q3. Auction Games (15 points + 10 points extra credit)

Consider an IPV setting where two bidders are risk-neutral and each bidder’s valuation is drawn from a uniform distribution over the range \([0, b]\).

- (15pts) Describe the optimal auction using virtual valuation and bidder-specific reserve price for this setting.

- (10pts extra credit) What is the seller’s expected revenue using this optimal auction mechanism, given bidders play the dominant strategy.