Sample Recurrence for Assignment 5

1 recall

The situation for this programming assignment is that we have

- coins of denominations $d_1, d_2, \ldots, d_n$
- a target value $T$
- where all denominations are distinct (though this is not important to the problem or solution)
- and 5 coins of each denomination

The problem is to write code that will determine the \textit{maximum} number of coins that will add up to the target value $T$ (exactly). Here we describe a recurrence relation to get you started on the code part.

2 subproblem

Define $MC(t, k)$ to be the maximum number of coins chosen from at most 5 each of $d_1, d_2, \ldots, d_k$ that add up to exactly $t$. The valid ranges of $t$ and $k$ are $0 \leq t \leq T$ and $0 \leq k \leq n$ (although for convenience below we allow $t < 0$ to be considered).

3 recurrence

The idea for inputs $t$ and $k$ (for coins $d_1, d_2, \ldots, d_k$) is to try all allowed amounts of coin $d_k$ and test the effects of that while using coins $d_1, d_2, \ldots, d_{k-1}$ on the remaining value (using $MC(*, k-1)$ recursively). If $i$ of the $d_k$ coins are used, the remaining target value is $t - i \cdot d_k$, so $i$ should be added to $MC(t - i \cdot d_k, k - 1)$.

$$MC(t, k) = \begin{cases} 0 & \text{if } t = 0 \\ -\infty & \text{if } t < 0 \\ -\infty & \text{if } k = 0 \text{ and } t > 0 \\ \max\{ i + MC(t - i \cdot d_k, k - 1) \mid 0 \leq i \leq 5 \} & \text{otherwise} \end{cases}$$
Want to return $MC(T, n)$. In this formulation it is not possible to get a set of coins to add up to $T$ if $MC(T, n) < 0$. 

4 target value