Office Hours: Weeks 4-10

• Monday: 1-2 (Roscoe)
• Tuesday: 1-2 (Roscoe)
• Wednesday: 1-3 (Roscoe)
• Thursday: 1130-1230 (Hank)
• Friday: 1130-1230 (Hank)

• All normal this week!!! 😊
Timeline

- **1C**: due Weds Jan 23rd
- **1D**: assigned today (LAST TUESDAY), due Thurs Jan 31st
- **1E**: assigned Thurs Jan 31st, due Weds Feb 6th
  - → will be extra support with this. Tough project.
- **1F**: assigned Feb 7th, due Feb 19th
  - → not as tough as 1E
- **2A**: will be assigned during week of Feb 11th

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Likely: pre-SuperBowl OH
Our goal

World space:
- Triangles in native Cartesian coordinates
- Camera located anywhere

Camera space:
- Camera located at origin, looking down -Z
- Triangle coordinates relative to camera frame

Image space:
- All viewable objects within -1 <= x,y,z <= +1

Screen space:
- All viewable objects within -1 <= x, y <= +1

Device space:
- All viewable objects within 0<=x<=width, 0 <=y<=height
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- All viewable objects within 0<=x<=width, 0 <=y<=height
World Space

- World Space is the space defined by the user’s coordinate system.
- This space contains the portion of the scene that is transformed into camera space by the camera transform.
- Many of the spaces have “bounds,” meaning limits on where the space is valid.
- With world space 2 options:
  - No bounds
  - User specifies the bound
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All viewable objects within
-1 <= x, y <= +1

Device space:
All viewable objects within
0 <= x <= width, 0 <= y <= height
How do we specify a camera?

The “viewing pyramid” or “view frustum”.

Frustum: In geometry, a frustum (plural: frusta or frustums) is the portion of a solid (normally a cone or pyramid) that lies between two parallel planes cutting it.

class Camera
{
    public:
        double near, far;
        double angle;
        double position[3];
        double focus[3];
        double up[3];
};
What is the up axis?

- Up axis is the direction from the base of your nose to your forehead.
What is the up axis?

- Up axis is the direction from the base of your nose to your forehead.
What is the up axis?

- Up axis is the direction from the base of your nose to your forehead.

- (If you lie down while watching TV, the screen is sideways.)
Image Space Diagram
Our goal

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Triangles in native Cartesian coordinates
Camera located anywhere

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Triangle coordinates relative to camera frame

Image space:
All viewable objects within
-1 <= x, y, z <= +1

Screen space:
All viewable objects within
-1 <= x, y <= +1

Device space:
All viewable objects within
0 <= x <= width, 0 <= y <= height

View Transform
Our goal

World space:
- Triangles in native Cartesian coordinates
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- Triangle coordinates relative to camera frame

Image space:
- All viewable objects within -1 <= x, y, z <= +1

Screen space:
- All viewable objects within -1 <= x, y <= +1

Device space:
- All viewable objects within 0 <= x <= width, 0 <= y <= height
Image Space

- Image Space is the three-dimensional coordinate system that contains screen space.
- It is the space where the view transformation directs its output.
- The bounds of Image Space are 3-dimensional cube.

\[ \{(x,y,z) : -1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1\} \]

(or \(-1 \leq z \leq 0\))
Image Space Diagram
Our goal

World space:
- Triangles in native Cartesian coordinates
- Camera located anywhere

Camera space:
- Camera located at origin, looking down -Z
- Triangle coordinates relative to camera frame

Image space:
- All viewable objects within
  -1 \leq x, y, z \leq +1

Screen space:
- All viewable objects within
  -1 \leq x, y \leq +1

Device space:
- All viewable objects within
  0 \leq x \leq \text{width}, 0 \leq y \leq \text{height}
Screen Space

- Screen Space is the intersection of the xy-plane with Image Space.

- Points in image space are mapped into screen space by projecting via a parallel projection, onto the plane $z = 0$.

- Example:
  - a point $(0, 0, z)$ in image space will project to the center of the display screen.
Screen Space Diagram
Our goal

World space:
- Triangles in native Cartesian coordinates
- Camera located anywhere

Camera space:
- Camera located at origin, looking down -Z
- Triangle coordinates relative to camera frame

Image space:
- All viewable objects within $-1 \leq x, y, z \leq +1$

Screen space:
- All viewable objects within $-1 \leq x, y \leq +1$

Device space:
- All viewable objects within $0 \leq x \leq \text{width}$, $0 \leq y \leq \text{height}$
Device Space

- Device Space is the lowest level coordinate system and is the closest to the hardware coordinate systems of the device itself.

- Device space is usually defined to be the $n \times m$ array of pixels that represent the area of the screen.

- A coordinate system is imposed on this space by labeling the lower-left-hand corner of the array as $(0,0)$, with each pixel having unit length and width.
Device Space Example

pixel (0, 0)

pixel (3, 7)

pixel (15, 15)
Extends Device Space to three dimensions by adding z-coordinate of image space.

Coordinates are \((x, y, z)\) with

\[
0 \leq x \leq n \\
0 \leq y \leq m \\
z \text{ arbitrary (but typically } -1 \leq z \leq +1 \text{ or } -1 \leq z \leq 0 \text{ )}
\]
Start Part 2 of YouTube Video

- In Part 2:
  - Device Space Transform
  - More Math Primer
How do we transform?

- For a camera $C$,
  - Calculate Camera Frame
  - From Camera Frame, calculate Camera Transform
  - Calculate View Transform
  - Calculate Device Transform
  - Compose 3 Matrices into 1 Matrix ($M$)

- For each triangle $T$, apply $M$ to each vertex of $T$, then apply rasterization/zbuffer

```cpp
class Camera {
public:
    double near, far;
    double angle;
    double position[3];
    double focus[3];
    double up[3];
};
```
Easiest Transform

World space:
- Triangles in native Cartesian coordinates
- Camera located anywhere

Camera space:
- Camera located at origin, looking down -Z
- Triangle coordinates relative to camera frame

Image space:
- All viewable objects within
  \(-1 \leq x, y, z \leq +1\)

Screen space:
- All viewable objects within
  \(-1 \leq x, y \leq +1\)

Device space:
- All viewable objects within
  \(0 \leq x \leq \text{width}, 0 \leq y \leq \text{height}\)
(x, y, z) \rightarrow (x', y', z'), where
\[ x' = \frac{n(x+1)}{2} = \frac{nx}{2} + \frac{n}{2} \]
\[ y' = \frac{m(y+1)}{2} = \frac{my}{2} + \frac{m}{2} \]
\[ z' = z = z \]
(for an $n \times m$ image)

Matrix:
\[
\begin{pmatrix}
 x & y & z & 1 \\
\end{pmatrix}
\begin{pmatrix}
 \frac{n}{2} & 0 & 0 & 0 \\
 0 & \frac{m}{2} & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 \frac{n}{2} & \frac{m}{2} & 0 & 1 \\
\end{pmatrix}
\]
Note: Ken Joy’s graphics notes are fantastic
What is the norm of a vector?

- The norm of a vector is its length
  - Denoted with $||\cdot||$

- For a vector $A = (A.x, A.y)$,
  $$||A|| = \sqrt{A.x * A.x + A.y * A.y}$$

- Physical interpretation:

- For 3D, $||A|| = \sqrt{A.x * A.x + A.y * A.y + A.z * A.z}$
What does it means for a vector to be normalized?

- The vector $\mathbf{A}$ is normalized if $\| \mathbf{A} \| = 1$.
  - This is also called a unit vector.
- To obtain a normalized vector, take $\mathbf{A}/\| \mathbf{A} \|$

- Many of the operations we will discuss today will only work correctly with normalized vectors.
What is a dot product?

- \[ \mathbf{A} \cdot \mathbf{B} = A.x \times B.x + A.y \times B.y \]
  - (or \( A.x \times B.x + A.y \times B.y + A.z \times B.z \))

- Physical interpretation:
  - \[ \mathbf{A} \cdot \mathbf{B} = \cos(\alpha) \times (||\mathbf{A}|| \times ||\mathbf{B}||) \]
What is the cross product?

- \( A \times B = (A.y \times B.z - A.z \times B.y, \)
  
  \( B.x \times A.z - A.x \times B.z, \)

- \( A.x \times B.y - A.y \times B.x) \)

- What is the physical interpretation of a cross product?
  - Finds a vector perpendicular to both A and B.
Homogeneous Coordinates

- Defined: a system of coordinates used in projective geometry, as Cartesian coordinates are used in Euclidean geometry
- Primary uses:
  - $4 \times 4$ matrices to represent general 3-dimensional transformations
  - It allows a simplified representation of mathematical functions – the rational form – in which rational polynomial functions can be simply represented
- We only care about the first
  - I don’t really even know what the second use means
Interpretation of Homogeneous Coordinates

- 4D points: \((x, y, z, w)\)
- Our typical frame: \((x, y, z, 1)\)
Interpretation of Homogeneous Coordinates

- 4D points: \((x, y, z, w)\)
- Our typical frame: \((x, y, z, 1)\)

So how to treat points not along the \(w=1\) line?

Our typical frame in the context of 4D points
Let $P = (x, y, z, w)$ be a 4D point with $w \neq 1$.

Goal: find $P' = (x', y', z', 1)$ such $P$ projects to $P'$.

(We have to define what it means to project)

Idea for projection:

- Draw line from $P$ to origin.
- If $Q$ is along that line (and $Q.w = 1$), then $Q$ is a projection of $P$. 

Projecting back to $w=1$ line
Projecting back to $w=1$ line

- **Idea for projection:**
  - Draw line from $P$ to origin.
  - If $Q$ is along that line (and $Q \cdot w = 1$), then $Q$ is a projection of $P$. 
So what is Q?

- Similar triangles argument:
  - $x' = x/w$
  - $y' = y/w$
  - $z' = z/w$
Our goal

World space:
Triangles in native Cartesian coordinates
Camera located anywhere

Camera space:
Camera located at origin, looking down -Z
Triangle coordinates relative to camera frame

- Need to construct a Camera Frame
- Need to construct a matrix to transform points from Cartesian Frame to Camera Frame
  - Transform triangle by transforming its three vertices
Camera frame must be a basis:

- Spans space ... can get any point through a linear combination of basis vectors

- Every member must be linearly independent
  - we didn’t talk about this much on Thursday.
  - linearly independent means that no basis vector can be represented via others
  - Repeat slide (coming up) shows linearly *dependent* vectors
(REPEAT) Why unique?

- Let \((a, b, c)\) mean:
  - The number of steps ‘a’ in direction D1
  - The number of steps ‘b’ in direction D2
  - The number of steps ‘c’ in direction D3

- Then there is more than one way to get to some point \(X\) in \(S\), i.e.,
  - \((a_1, b_1, c_1) = X\) and
  - \((a_2, b_2, c_2) = X\)
Camera frame construction

- **Must choose** \((u,v,w,O)\)

- **O** = camera position

- **w** = O-focus

  - Not “focus-O”, since we want to look down -Z

```cpp
class Camera {
    public:
        double near, far;
        double angle;
        double position[3];
        double focus[3];
        double up[3];
};
```
Camera frame construction

- Must choose \((u, v, w, O)\)

Camera space:
- Camera located at origin, looking down -Z
- Triangle coordinates relative to camera frame

- \(O\) = camera position
- \(w\) = \(O\)-focus
- \(v\) = up
- \(u\) = up \(\times\) (\(O\)-focus)

```cpp
class Camera {
public:
    double near, far;
    double angle;
    double position[3];
    double focus[3];
    double up[3];
};
```
But wait ... what if \( \text{dot}(v_2,v_3) \neq 0 \)?

- We can get around this with two cross products:
  - \( u = \text{Up} \times (O\text{-focus}) \)
  - \( v = (O\text{-focus}) \times u \)
Camera frame summarized

- $O =$ camera position
- $u =$ Up x (O-focus)
- $v =$ (O-focus) x $u$
- $w =$ O-focus

Important note: $u$, $v$, and $w$ need to be normalized!
Our goal

- Need to construct a Camera Frame
- Need to construct a matrix to transform points from Cartesian Frame to Camera Frame
  - Transform triangle by transforming its three vertices
Consider the meaning of Cartesian coordinates \((x,y,z)\):

\[
\begin{bmatrix}
x & y & z & 1
\end{bmatrix}
\begin{bmatrix}
<1,0,0>
\end{bmatrix}
\]

\[
\begin{bmatrix}
<0,1,0>
\end{bmatrix} = (x,y,z)
\]

\[
\begin{bmatrix}
<0,0,1>
\end{bmatrix}
\]

\[(0,0,0)\]
The Two Frames of the Camera Transform

☐ Our two frames:

☐ Cartesian:
  ☐ <1,0,0>
  ☐ <0,1,0>
  ☐ <0,0,1>
  ☐ (0,0,0)

☐ Camera:
  ☐ u = up x (O-focus)
  ☐ v = (O-focus) x u
  ☐ w = (O-focus)
  ☐ O
The Two Frames of the Camera Transform

- Our two frames:
  - Cartesian:
    - <1,0,0>
    - <0,1,0>
    - <0,0,1>
    - (0,0,0)
  - Camera:
    - u = up x (O-focus)
    - v = (O-focus) x u
    - w = (O-focus)
    - O

The “Camera Frame” is a Frame, so we can express any Cartesian vector as a combination of u, v, w.
Converting From Cartesian Frame To Camera Frame

- The Cartesian vector \( <1,0,0> \) can be represented as some combination of the Camera Frame’s basis functions \( u, v, w: \)
  \[ <1,0,0> = e_{1,1} * u + e_{1,2} * v + e_{1,3} * w \]

- So can the Cartesian vector \( <0,1,0> \)
  \[ <0,1,0> = e_{2,1} * u + e_{2,2} * v + e_{2,3} * w \]

- So can the Cartesian vector \( <0,0,1> \)
  \[ <0,0,1> = e_{3,1} * u + e_{3,2} * v + e_{3,3} * w \]

- So can the vector: Cartesian Frame origin – Camera Frame origin
  \[ (0,0,0) - O = e_{4,1} * u + e_{4,2} * v + e_{4,3} * w \Rightarrow \]
  \[ (0,0,0) = e_{4,1} * u + e_{4,2} * v + e_{4,3} * w + O \]
Putting Our Equations Into Matrix Form

\[ <1,0,0> = e_{1,1} \cdot u + e_{1,2} \cdot v + e_{1,3} \cdot w \]

\[ <0,1,0> = e_{2,1} \cdot u + e_{2,2} \cdot v + e_{2,3} \cdot w \]

\[ <0,0,1> = e_{3,1} \cdot u + e_{3,2} \cdot v + e_{3,3} \cdot w \]

\[ (0,0,0) = e_{4,1} \cdot u + e_{4,2} \cdot v + e_{4,3} \cdot w + O \]

\[ \rightarrow \]

\[ [<1,0,0>] = \begin{bmatrix} e_{1,1} & e_{1,2} & e_{1,3} & 0 \end{bmatrix} [u] \]

\[ [<0,1,0>] = \begin{bmatrix} e_{2,1} & e_{2,2} & e_{2,3} & 0 \end{bmatrix} [v] \]

\[ [<0,0,1>] = \begin{bmatrix} e_{3,1} & e_{3,2} & e_{3,3} & 0 \end{bmatrix} [w] \]

\[ (0,0,0) = \begin{bmatrix} e_{4,1} & e_{4,2} & e_{4,3} & 1 \end{bmatrix} [O] \]
Here Comes The Trick…

- Consider the meaning of Cartesian coordinates \((x,y,z)\):

\[
\begin{bmatrix}
  x & y & z & 1
\end{bmatrix}
\begin{bmatrix}
  <1,0,0> \\
  <0,1,0> \\
  <0,0,1> \\
  (0,0,0)
\end{bmatrix} = (x, y, z)
\]

But:

\[
\begin{bmatrix}
  <1,0,0> \\
  <0,1,0> \\
  <0,0,1> \\
  (0,0,0)
\end{bmatrix}
\begin{bmatrix}
  e_{1,1} & e_{1,2} & e_{1,3} & 0 \\
  e_{2,1} & e_{2,2} & e_{2,3} & 0 \\
  e_{3,1} & e_{3,2} & e_{3,3} & 0 \\
  e_{4,1} & e_{4,2} & e_{4,3} & 1
\end{bmatrix}
\begin{bmatrix}
  u \\
  v \\
  w \\
  0
\end{bmatrix}
\]
Here Comes The Trick…

But:

\[
\begin{bmatrix}
<1,0,0> \\
<x, y, z, 1> \\
<0,0,1> \\
(0,0,0)
\end{bmatrix}
\]

\[
\begin{bmatrix}
[e1,1 & e1,2 & e1,3 & 0] \\
[x & y & z & 1] \\
[e2,1 & e2,2 & e2,3 & 0] \\
[e3,1 & e3,2 & e3,3 & 0] \\
[e4,1 & e4,2 & e4,3 & 1]
\end{bmatrix}
\]

Coordinates of \((x,y,z)\) with respect to Cartesian frame.

Coordinates of \((x,y,z)\) with respect to Camera frame.

So this matrix is the camera transform!!
And Cramer’s Rule Can Solve This, For Example…

\[ e_{1,1} = \frac{(\langle 1,0,0 \rangle \times \vec{v}) \cdot \vec{w}}{(\vec{u} \times \vec{v}) \cdot \vec{w}} \]

\[ e_{1,2} = \frac{(\vec{u} \times \langle 1,0,0 \rangle) \cdot \vec{w}}{(\vec{u} \times \vec{v}) \cdot \vec{w}} \]

\[ e_{1,3} = \frac{(\vec{u} \times \vec{v}) \cdot \langle 1,0,0 \rangle}{(\vec{u} \times \vec{v}) \cdot \vec{w}} \]

(u == v1, v == v2, w == v3 from previous slide)
Solving the Camera Transform

\[
\begin{bmatrix}
e1,1 & e1,2 & e1,3 & 0 \\
e2,1 & e2,2 & e2,3 & 0 \\
e3,1 & e3,2 & e3,3 & 0 \\
e4,1 & e4,2 & e4,3 & 1 \\
\end{bmatrix} = \begin{bmatrix}
u.x & v.x & w.x & 0 \\
u.y & v.y & w.y & 0 \\
u.z & v.z & w.z & 0 \\
u.t & v.t & w.t & 1 \\
\end{bmatrix}
\]

Where \( t = (0,0,0)-O \)

How do we know?: Cramer’s Rule + simplifications

Want to derive?:

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Device space:
- All viewable objects within 0 <= x <= width, 0 <= y <= height
The viewing transformation is not a combination of simple translations, rotations, scales or shears: it is more complex.
I personally don’t think it is a good use of class time to derive this matrix.

Well derived at:

The View Transformation

- Input parameters: \((\alpha, n, f)\)
- Transforms view frustum to image space cube
  - View frustum: bounded by viewing pyramid and planes \(z=-n\) and \(z=-f\)
  - Image space cube: \(-1 \leq u,v,w \leq 1\)
    
    \[
    \begin{bmatrix}
    \cot(\alpha/2) & 0 & 0 & 0 \\
    0 & \cot(\alpha/2) & 0 & 0 \\
    0 & 0 & (f+n)/(f-n) & -1 \\
    0 & 0 & 2fn/(f-n) & 0
    \end{bmatrix}
    \]
  - Cotangent = \(1/\text{tangent}\)
Let’s do an example

- **Input parameters**: \((\alpha, n, f) = (90, 5, 10)\)

\[
\begin{bmatrix}
\cot(\alpha/2) & 0 & 0 & 0 & 0 \\
0 & \cot(\alpha/2) & 0 & 0 \\
0 & 0 & (f+n)/(f-n) & -1 \\
0 & 0 & 2fn/(f-n) & 0
\end{bmatrix}
\]
Let’s do an example

- Input parameters: \((\alpha, n, f) = (90, 5, 10)\)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 3 & -1 & 0 \\
0 & 0 & 20 & 0 & 0 \\
\end{bmatrix}
\]
Let’s do an example

- **Input parameters:** \((\alpha, n, f) = (90, 5, 10)\)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 3 & -1 \\
0 & 0 & 20 & 0
\end{bmatrix}
\]

Let’s multiply some points:
- \((0,7,-6,1)\)
- \((0,7,-8,1)\)
Let’s do an example

**Input parameters:** \((\alpha, n, f) = (90, 5, 10)\)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 3 & -1 \\
0 & 0 & 20 & 0
\end{bmatrix}
\]

Let’s multiply some points:

\((0,7,-6,1) = (0,7,2,6) = (0, 1.16, 0.33)\)
\((0,7,-8,1) = (0,7,-4,8) = (0, 0.88, -0.5)\)
Let's do an example

- **Input parameters:** \((\alpha, n, f) = (90, 5, 10)\)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 3 & -1 \\
0 & 0 & 20 & 0
\end{bmatrix}
\]

More points:
- \((0,7,-4,1) = (0,7,8,4) = (0, 1.75, 2)\)
- \((0,7,-5,1) = (0,7,5,5) = (0, 1.4, 1)\)
- \((0,7,-6,1) = (0,7,2,6) = (0, 1.16, 0.33)\)
- \((0,7,-8,1) = (0,7,-4,8) = (0, 0.88, -0.5)\)
- \((0,7,-10,1) = (0,7,-10,10) = (0, 0.7, -1)\)
- \((0,7,-11,1) = (0,7,-13,11) = (0, .63, -1.18)\)
The viewing transformation is not a combination of simple translations, rotations, scales or shears: it is more complex.
More points:
(0,7,-4,1) = (0,7,8,4) = (0, 1.75, 2)
(0,7,-5,1) = (0,7,5,5) = (0, 1.4, 1)
(0,7,-6,1) = (0,7,2,6) = (0, 1.16, 0.33)
(0,7,-8,1) = (0,7,-4,8) = (0, 0.88, -0.5)
(0,7,-10,1) = (0,7,-10,10) = (0, 0.7, -1)
(0,7,-11,1) = (0,7,-13,11) = (0, .63, -1.18)

Note there is a non-linear relationship in W (“Z”).
Putting It All Together
How do we transform?

- For a camera $C$,
  - Calculate Camera Frame
  - From Camera Frame, calculate Camera Transform
  - Calculate View Transform
  - Calculate Device Transform
  - Compose 3 Matrices into 1 Matrix ($M$)

- For each triangle $T$, apply $M$ to each vertex of $T$, then apply rasterization/depth buffer

```cpp
class Camera {
    public:
        double near, far;
        double angle;
        double position[3];
        double focus[3];
        double up[3];
};
```
Goal: add arbitrary camera positions

Extend your project1D code

New: proj1e_geometry.vtk available on web (9MB), "reader1e.cxx".

New: Matrix.cxx, Camera.cxx

No Cmake, project1E.cxx
Project #1E, expanded

- Matrix.cxx: complete

- Methods:

```cpp
class Matrix
{
    public:
        double A[4][4];

    void TransformPoint(const double *ptIn, double *ptOut);
    static Matrix ComposeMatrices(const Matrix &, const Matrix &);
    void Print(ostream &o);
};
```
Camera.cxx: you work on this

```cpp
class Camera {
    public:
        double          near, far;
        double          angle;
        double          position[3];
        double          focus[3];
        double          up[3];

        Matrix          ViewTransform(void) {;};
        Matrix          CameraTransform(void) {;};
        Matrix          DeviceTransform(void) {;};
        // Will probably need something for calculating Camera Frame as well

    }

Also: GetCamera(int frame, int nFrames)
```
Project #1E, deliverables

- Same as usual, but times 4
  - 4 images, corresponding to
    - GetCamera(0, 1000)
    - GetCamera(250,1000)
    - GetCamera(500,1000)
    - GetCamera(750,1000)

- If you want:
  - Generate all thousand images, make a movie
    - Then you should wait for 1F. Then we will have shading too.
vector<Triangle> t = GetTriangles();
AllocateScreen();
for (int i = 0 ; i < 4 ; i++)
{   int f = 250*i;
    InitializeScreen();
    Camera c = GetCamera(f, 1000);
    TransformTrianglesToDeviceSpace(); // involves setting up and applying matrices
    //... if you modify vector<Triangle> t,
    // remember to undo it later
    RenderTriangles();
    SaveImage();
}
Correct answers given for GetCamera(0, 1000)

Camera Frame: U = 0, 0.707107, -0.707107
Camera Frame: V = -0.816497, 0.408248, 0.408248
Camera Frame: W = 0.57735, 0.57735, 0.57735
Camera Frame: O = 40, 40, 40

Camera Transform
(0.0000000 -0.8164966 0.5773503 0.0000000)
(0.7071068 0.4082483 0.5773503 0.0000000)
(-0.7071068 0.4082483 0.5773503 0.0000000)
(0.0000000 0.0000000 -69.2820323 1.0000000)

View Transform
(3.7320508 0.0000000 0.0000000 0.0000000)
(0.0000000 3.7320508 0.0000000 0.0000000)
(0.0000000 0.0000000 1.0512821 -1.0000000)
(0.0000000 0.0000000 10.2564103 0.0000000)

Transformed 37.1132, 37.1132, 37.1132, 1 to 0, 0, 1
Transformed -75.4701, -75.4701, -75.4701, 1 to 0, 0, -1
Project #1E pitfalls

- All vertex multiplications use 4D points. Make sure you send in 4D points for input and output, or you will get weird memory errors.
  - Make sure you divide by w.
People often get a matrix confused with its transpose. Use the method Matrix::Print() to make sure the matrix you are setting up is what you think it should be. Also, remember the points are left multiplied, not right multiplied.

Regarding multiple renderings:

- Don’t forget to initialize the screen between each render
- If you modify the triangle in place to render, don’t forget to switch it back at the end of the render
Project #1F (8%), Due Feb 19th

- Goal: add shading, movie