\[ a < b \quad a \geq b \quad a \geq b+1 \]

**Before insertion**
\[ a \quad b \]

**After**
\[ a = a' \quad b' = b + 1 \]

Show: \( a' \) can't be \( b' \)

\[ a' \neq b' \]

\[ a \leq b \quad (\text{otherwise the height of the subtree would increase after insertion}) \]

\[ a < b + 1 \]

\[ a' < b' \]
Converting 2-3-4 tree to red-black tree
black-height of a node:

is # back node from it to some leaf (excluding the node itself).

If a node has n nodes in the subtree rooted at x

\[ \text{bh}(x) \geq n - 1 \]

by induction on n

- \( n = 1 \):
  \[ \text{bh}(x) \geq 2 - 1 = 1 \]

- \( n > 1 \):
  \[ \text{bh}(x) \geq 2 \text{ bh}(x) - 1 \]

\[ \text{bh}(x) \geq 2 \text{ bh}(x) - 1 \]

\[ \text{bh}(x) = 0 \]

\[ \text{bh}(x) - 1 \]

\[ \text{bh}(x) - 1 \]

\[ \text{bh}(x) - 1 \]

\[ \text{bh}(x) - 1 \]
A tree of height $h$ has:

- At least $b h(\text{root}) \geq \frac{h}{2}$

For $n > 2$, $h - 1 \geq n - 1$

So,

\[
\frac{h}{2} \leq \log(n+1) - h \leq 2\log(n+1)
\]