CSE 473/573: Probabilistic Methods in AI

Homework #2

Submit via Gradescope by Thursday, April 19, 2018 at 11:00pm

Guidelines: You can brainstorm with others, but please solve the problems and write up the answers by yourself. Please show enough of your work to make your approach clear.

1. Consider the following Bayesian network structure:

   \[
   \begin{align*}
   &A \\
   &C \quad \uparrow \quad B \\
   &\quad \quad D \quad \quad \downarrow \\
   &E
   \end{align*}
   \]

   (a) What is the Markov blanket of A?
   (b) Is C independent of B? Why or why not? Explain.
   (c) Is C independent of D given B? Why or why not? Explain.
   (d) Is B independent of E given D? Why or why not? Explain.

2. 1 person in 10,000 has Huntington’s disease, a genetic disorder that usually manifests itself in middle age. Huntington’s disease is caused by an autosomal dominant mutation. This means that the probability of a child inheriting the disease from parents is 0% if neither parent has Huntington’s disease, 50% if one parent has it, and 75% if both parents have Huntington’s disease. (We ignore rare mutations and the extremely small fraction of people that have two copies of the defective gene.)

   (a) Consider a family consisting of a mother, a father, a son, and a daughter. Construct a Bayesian network representing the probability that each has Huntington’s disease. Use the binary variables M (Mother), F (Father), S (Son), and D (Daughter) to represent whether or not each person has Huntington’s disease.

   Provide both the structure \textbf{and} conditional probability distributions for this BN.

   (b) Prove or disprove the following independence and conditional independence assertions, based on your BN.

   If the assertion is true, prove it using the \textbf{structure of the network}.

   If the assertion is false, disprove it by \textbf{computing actual probability values} and showing that they do not satisfy the definition of conditional independence. (For convenience, you may simplify the arithmetic by rounding numbers slightly, e.g.: 1/10,000 + 1/100,000,000 \approx 1/10,000.)

   \textbullet\ \ M \perp F
   \textbullet\ \ M \perp F | S
• $S \perp D$

3. K&F 3.4 (Other types of intercausal reasoning.) HINT: I highly recommend that you write a spreadsheet or a simple program to test probabilities for this problem, so that you can try out several ideas and check them quickly.

4. [Grads only] K&F 3.10 (Prove that a node is d-separated from its non-descendants given its parents.)