CIS 473/573: Probabilistic Methods in AI

Homework #1

Submit via Gradescope by Thursday, April 12, 2018 at 11:00pm

Guidelines: You can brainstorm with others, but please write up the answers by yourself. You may use textbooks (Koller & Friedman, Russell & Norvig, etc.), your notes, and lecture slides.

Please show enough of your work to make your approach clear. Answers should be typewritten whenever possible. When submitting to Gradescope, be sure to select the right pages for each question.

1. (9 points) Given the following tables defining $P(A|B, C)$, $P(B)$, and $P(C|B)$, compute $P(A, B, C)$, $P(A, C)$, and $P(C|A)$. Express your answers as tables in the same format.

   \[
   \begin{array}{ccc}
   A & B & C & P(A|B, C) \\
   a^0 & b^0 & c^0 & 0.9 \\
   a^0 & b^0 & c^1 & 0.5 \\
   a^0 & b^1 & c^0 & 0.3 \\
   a^0 & b^1 & c^1 & 0.8 \\
   a^1 & b^0 & c^0 & 0.1 \\
   a^1 & b^0 & c^1 & 0.5 \\
   a^1 & b^1 & c^0 & 0.7 \\
   a^1 & b^1 & c^1 & 0.2 \\
   \end{array}
   \]

   \[
   \begin{array}{cc}
   B & P(B) \\
   b^0 & 0.6 \\
   b^1 & 0.4 \\
   \end{array}
   \]

   \[
   \begin{array}{cc}
   C & P(C|B) \\
   c^0 & b^1 & 0.6 \\
   c^1 & b^0 & 0.8 \\
   \end{array}
   \]

   HINT: Use a calculator. When computing $P(A, B, C)$, you should obtain $P(a^0, b^0, c^0) = 0.9 \cdot 0.6 \cdot 0.2 = 0.108$ and $P(a^1, b^1, c^0) = 0.7 \cdot 0.4 \cdot 0.6 = 0.168$ as two of the eight entries. If you do this step correctly, then the entries in $P(A, B, C)$ should sum to one without requiring any additional normalization. If you get confused, ask for help!

2. (5 points) Suppose the false positive rate of an antibody test used to diagnose HIV is 5/100,000, and that the false negative rate is negligible (you may assume it to be zero). In this context, “false positive rate” is the probability of a person who does not have HIV wrongly testing positive for HIV, and “false negative rate” is the probability of a person with HIV wrongly testing negative for HIV. Suppose that, out of 300 million people living in the US, 1 million have HIV. If a random person in the US is given this HIV test and receives a positive result, what is the probability that they actually have HIV?
3. (6 points) K&F 2.3: Consider two events $\alpha$ and $\beta$ such that $P(\alpha) = p_a$ and $P(\beta) = p_b$. Given only that knowledge, what is the maximum and minimum values of the probability of the events $\alpha \cap \beta$ (“$\alpha$ AND $\beta$”) and $\alpha \cup \beta$ (“$\alpha$ OR $\beta$”)? Characterize the situations in which each of these extreme values occurs.

**HINTS:** Express your answers in terms of $p_a$ and $p_b$. Give the tightest bounds you can. Your answers should hold for any value of $p_a$ and $p_b$.

4. (12 points) Koller & Friedman, Exercise 2.10. *Graduate students must fully justify the answers for any case where the numbers are insufficient. Undergraduates do not.* This question investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations. Let $H, E_1, E_2$ be three random variables.

(a) Suppose we wish to calculate $P(H|E_1, E_2)$, and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

- $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$.
- $P(E_1, E_2), P(H)$, and $P(E_1, E_2|H)$.
- $P(E_1|H), P(E_2|H)$, and $P(H)$.

For each case justify your response either by showing how to calculate the desired answer from the numbers given, or (for graduate students) by explaining why this is not possible.

(b) Suppose we know that $E_1$ and $E_2$ are conditionally independent given $H$. Now which of the preceding three sets is sufficient? Justify your response as before.

5. (6 points) [Grads only] K&F 2.2:

- Show that for binary random variables $X,Y$, the event-level independence $(x^0 \perp y^0)$ implies random-variable independence $(X \perp Y)$.
- Show a counterexample for nonbinary variables.
- Is it the case that, for a binary-valued variable $Z$, we have that $(X \perp Y|z^0)$ implies $(X \perp Y|Z)$? (Explain.)