CIS 451/551: B+ tree insertion

related to a homework 5 problem
text’s rules (Ramakrishnan-Gehrke)

• d is the **order** of the tree (fixed up front)

• **internal node** of order d structure:
  \[ <P_0, K_1, P_1, K_2, ..., P_{m-1}, K_m, P_m> \]

• **leaf node**:
  \[ <P_{\text{prev}}, (K_1, P_1), (K_2, P_2), ..., (K_m, P_m), P_{\text{next}}> \]

• for \( d \leq m \leq 2d \)

• except root allows \( 1 \leq m \leq 2d \)
right bias on splitting

overflowing leaf node split

(a b c d e)

becomes

(c a b)

promote c by inserting into parent

overflowing internal node split

(a b c d e)

becomes

(c a d e)

similarly c gets promoted

left “bias” would have nodes (abc) and (de), which is a bit more common

promoted value remains in leaf node, does not remain in internal node
now the problem

insert into empty B+ tree with order d=2 the values:

20, 40, 37, 27, 25, 30, 15, 60, 42, 35, 5, 22, 50, 11, 32, 36
recall with order d=2

• leaf node can have 2-4 keys, with 2-4 child pointers
• internal node can have 2-4 keys with 3-5 child pointers
• the root is an exception: it can have a single key
• we are using the B+ tree “rules” from our text
• so on search, “ties” go to the right
• other texts **will** have slightly different rules so beware
starting to insert

after 20: (20)

after 40: (20, 40)

after 37: (20, 37, 40)

after 27: (20, 27, 37, 40)
after 25 overflow, so need to split:

\[(20, 25, 27, 37, 40)\]  

then inserting 30 and 15 is boring:

\[(27)\]  
\[(15, 20, 25)\]  
\[(27, 30, 37, 40)\]
after 60 overflow again:

- (27)
  - (15, 20, 25)
  - (27, 30, 37, 40, 60)

then inserting 42, 35, 5 is simple:

- (27, 37)
  - (5, 15, 20, 25)
  - (27, 30, 35)
  - (37, 40 42, 60)
inserting 22 causes a split:

(27, 37)  
(5, 15, 20, 22, 25)  (27, 30, 35)  (37, 40 42, 60)

(20, 27, 37)
(5, 15)  (20, 22, 25)  (27, 30, 35)  (37, 40, 42, 60)
50 also causes a split:

- (20, 27, 37)
  - (5, 15)
  - (20, 22, 25)
  - (27, 30, 35)
  - (37, 40, 42, 50, 60)

becoming

- (20, 27, 37, 42)
  - (5, 15)
  - (20, 22, 25)
  - (27, 30, 35)
  - (37, 40)
  - (42, 50, 60)
inserting 11 and 32 again simple

note that many of the insertions involve little work
now inserting 36 will cause two splits

32 is promoted into parent, but then the root node overflows, continued on next page
final tree

note
• the root may have only one key (and two children)
• the leaf nodes are linked together (not shown here)