1. Let \( f(n) = 313 \cdot n^9 + 314 \cdot n^8(\lg n)^2 + 212 \cdot n^3 + 315 \). Indicate whether each of the following statements is true or false.

(a) \( f(n) = O(2^n) \)

(b) \( f(n) = \Omega(2^n) \)

(c) \( f(n) = O(n^9) \)

(d) \( f(n) = O(n^8(\lg n)^2) \)

(e) \( f(n) = \Theta(n^9) \)

(f) \( f(n) = \Theta(n^9 \lg n) \)

(g) \( f(n) = \Omega(n^3) \)

(h) \( n^3 = \Omega(f(n)) \)
2. The Build-Max-Heap method when run on an array of \( n \) elements will convert it into a max-heap.

(a) What is the time complexity of this algorithm? Give the tightest available bound.

(b) Illustrate Build-Max-Heap on this array of \( n = 10 \) items. You may represent the heap as an either an array or a tree. Show intermediate results after each call to MAX-HEAPIFY.

\[
\begin{array}{cccccccccc}
5 & 7 & 12 & 8 & 2 & 10 & 6 & 4 & 9 & 11 \\
\end{array}
\]
3. Given a BST $T$ we want to perform a *level order* of $T$: print the contents of all nodes at depth 0, then all nodes at depth 1, depth 2, etc (each level should be printed from left to right). The root of the tree is $T\.root$ and a node $p$ has fields $p\.value$, $p\.lchild$, and $p\.rchild$. Give pseudo-code to perform a level-order traversal of $T$, printing the value fields of each node. Use the *queue* methods *enqueue* and *dequeue*.
4. Consider the following piece of code which, on input $x$ and $n$ (both non-negative integers), returns $x^{2^n}$:

```python
p = x
i = 0
while (i < n):
    p = p*p
    i = i+1
return p
```

You are to prove that this code does indeed return $x^{2^n}$ by the method of loop invariant. An invariant that should work here is $S(p, i)$, which is true iff

$$(i \leq n) \text{ and } (p = x^{2^i})$$

**Initialization** Show that prior to entry into the loop, $S(p, i) = S(x, 0)$ is true.

**Maintanance** Show that if $S(p, i)$ is true and the loop is entered (so $i < n$), then $S(p', i')$ is true after one more iteration of the loop ($p', i'$ are the new values given to $p, i$ in the loop body).

**Termination** Show that when exiting the loop, $p$ contains the desired value $x^{2^n}$.