1. Let \( f(n) = 14n^4 + 5n^3 + 2n \log n + 15 \). Indicate whether each of the following statements is true or false.

   (a) \( f(n) = O(2^n) \)
   (b) \( f(n) = O(n^4) \)
   (c) \( f(n) = o(n^4) \)
   (d) \( f(n) = 14n^4 + o(n^4) \)
   (e) \( f(n) = \Theta(n^3) \)
   (f) \( f(n) = \Omega(n^3) \)

   [6 points]

2. Into an initially empty AVL tree, insert the following values:

   \( 10, 4, 12, 3, 2, 5, 6, 7, 8, 9 \).

   [10 points]

3. Insert the values above into an initially empty 2-3-4 tree. [10 points]

4. Into an initially empty red-black tree, insert the following values:

   \( 12, 8, 5, 10, 15, 17 \).

   [6 points]

5. What are the run-times of the following pieces of code?

   (a) for \( i = 1 \) to \( n \times n \)
       \( j = 1 \)
       while \( (j < i) \) {
           sum ++
           j = j * 3
       }

   (b) for \( i = 1 \) to \( n \)
       for \( j = 1 \) to \( i \times i \)
           sum++

   [8 points]

TURN THE PAGE OVER FOR QUESTION 6
6. Suppose we have a BST $T$, with root $T.root$, and $n$ nodes. Each node $p$ of the tree has fields $p.lchild$, $p.rchild$, $p.value$, and $p.bf$, where the last stands for balance factor. This balance factor of node $p$ is defined to be $height(p.lchild)-height(p.rchild)$. All the fields of each node have been filled except for this balance factor, which is empty.

Write a $\Theta(n)$ time method (in pseudo-code) which will calculate the balance factors of all the nodes of the tree, and store the balance factor of node $p$ in the $p.bf$ field.

You do not have access to a height function (*hint: write one and modify it*). It is possible the tree is not an AVL tree - you do not need to check whether it is. [12 points]

Total: 52 points