CIS 473/573: Probabilistic Methods in AI

Homework #6 – Written
Due via Gradescope at 11:00pm on Monday, May 22, 2017

Guidelines: You can brainstorm with others, but please solve the problems and write up the answers by yourself. You may use textbooks (Koller & Friedman, Russell & Norvig, etc.), your notes, and lecture slides. Please do NOT use any other resources (e.g., online problem solutions) without asking.

Please show enough of your work to make your approach clear.

1. Consider the following Markov network:

With the following factors:

\[
\begin{align*}
\phi_{1}(A, D) & : a^0 d^0 \quad a^0 d^1 \quad a^1 d^0 \quad a^1 d^1 \\
\phi_{2}(B, D) & : b^0 d^0 \quad b^0 d^1 \quad b^1 d^0 \quad b^1 d^1 \\
\phi_{1}(C, D) & : c^0 d^0 \quad c^0 d^1 \quad c^1 d^0 \quad c^1 d^1 \\
\phi_{1}(D, E) & : d^0 e^0 \quad d^0 e^1 \quad d^1 e^0 \quad d^1 e^1 \\
\phi_{1}(E, F) & : e^0 f^0 \quad e^0 f^1 \quad e^1 f^0 \quad e^1 f^1 \\
\end{align*}
\]

(a) Manually perform variable elimination to compute the partition function, \(Z\).

Show all of the intermediate factors along with a few sample computations to demonstrate how you compute them.

(b) Which variable should **not** be eliminated first, and why?

2. [Grads only] Consider a Markov network consisting of a single loop of pairwise potentials:

\[
P(X_1, \ldots, X_n) = \frac{1}{Z} \phi_1(X_1, X_2) \phi_2(X_2, X_3) \cdots \phi_{n-1}(X_{n-1}, X_n) \phi_n(X_n, X_1)
\]

(a) Show that eliminating any variable \(X_i\) will yield an intermediate factor with a scope of three variables.

(b) Use your result from part a to prove that eliminating all variables from any network with cycles is \(\Omega(|Val(X_i)|^3)\), where \(|Val(X_i)|\) is the minimum number of values of any variable \(X_i \in X\). (Recall that Big-\(\Omega\) is analogous to Big-\(O\), but a lower bound instead of an upper bound.)