Grading

- **Undergraduate**
  - 5% homework
  - 10% pattern programming labs
  - 15% individual programming assignment
  - 35% midterm exam
  - 35% project

- **Graduate**
  - 15% individual programming assignment
  - 30% midterm exam
  - 35% project
  - 20% research paper
Methodological Design

- Partition
  - Task/data decomposition

- Communication
  - Task execution coordination

- Agglomeration
  - Evaluation of the structure

- Mapping
  - Resource assignment

I. Foster, “Designing and Building Parallel Programs,” Addison-Wesley, 1995. Book is online, see webpage.
Partitioning

- Partitioning stage is intended to expose opportunities for parallel execution
- Focus on defining large number of small task to yield a fine-grained decomposition of the problem
- A good partition divides into small pieces both the computational tasks associated with a problem and the data on which the tasks operates
- *Domain decomposition* focuses on computation data
- *Functional decomposition* focuses on computation tasks
- Mixing domain/functional decomposition is possible
Domain and Functional Decomposition

- Domain decomposition of 2D / 3D grid

- Functional decomposition of a climate model
Partitioning Checklist

- Does your partition define at least an order of magnitude more tasks than there are processors in your target computer? If not, may lose design flexibility.

- Does your partition avoid redundant computation and storage requirements? If not, may not be scalable.

- Are tasks of comparable size? If not, it may be hard to allocate each processor equal amounts of work.

- Does the number of tasks scale with problem size? If not may not be able to solve larger problems with more processors

- Have you identified several alternative partitions?
Communication (Interaction)

- Tasks generated by a partition must interact to allow the computation to proceed
  - Information flow: data and control

- Types of communication
  - *Local* vs. *Global*: locality of communication
  - *Structured* vs. *Unstructured*: communication patterns
  - *Static* vs. *Dynamic*: determined by runtime conditions
  - *Synchronous* vs. *Asynchronous*: coordination degree

- Granularity and frequency of communication
  - Size of data exchange

- Think of communication as interaction and control
  - Applicable to both shared and distributed memory parallelism
Types of Communication

- Point-to-point
- Group-based
- Hierarchical
- Collective
Communication Design Checklist

- Is the distribution of communications equal?
  - Unbalanced communication may limit scalability
- What is the communication locality?
  - Wider communication locales are more expensive
- What is the degree of communication concurrency?
  - Communication operations may be parallelized
- Is computation associated with different tasks able to proceed concurrently? Can communication be overlapped with computation?
  - Try to reorder computation and communication to expose opportunities for parallelism
Agglomeration

- Move from parallel abstractions to real implementation
- Revisit partitioning and communication
  - View to efficient algorithm execution
- Is it useful to agglomerate?
  - What happens when tasks are combined?
- Is it useful to replicate data and/or computation?
- Changes important algorithm and performance ratios
  - Surface-to-volume: reduction in communication at the expense of decreasing parallelism
  - Communication/computation: which cost dominates
- Replication may allow reduction in communication
- Maintain flexibility to allow overlap
Types of Agglomeration

- Element to column

- Element to block
  - Better surface to volume

- Task merging

- Task reduction
  - Reduces communication
Agglomeration Design Checklist

- Has increased locality reduced communication costs?
- Is replicated computation worth it?
- Does data replication compromise scalability?
- Is the computation still balanced?
- Is scalability in problem size still possible?
- Is there still sufficient concurrency?
- Is there room for more agglomeration?
- Fine-grained vs. coarse-grained?
Mapping

- Specify where each task is to execute
  - Less of a concern on shared-memory systems

- Attempt to minimize execution time
  - Place concurrent tasks on different processors to enhance physical concurrency
  - Place communicating tasks on same processor, or on processors close to each other, to increase locality
  - Strategies can conflict!

- Mapping problem is \textit{NP-complete}
  - Use problem classifications and heuristics

- Static and dynamic load balancing
Mapping Algorithms

- Load balancing (partitioning) algorithms
- Data-based algorithms
  - Think of computational load with respect to amount of data being operated on
  - Assign data (i.e., work) in some known manner to balance
  - Take into account data interactions
- Task-based (task scheduling) algorithms
  - Used when functional decomposition yields many tasks with weak locality requirements
  - Use task assignment to keep processors busy computing
  - Consider centralized and decentralize schemes
Mapping Design Checklist

- Is static mapping too restrictive and non-responsive?
- Is dynamic mapping too costly in overhead?
- Does centralized scheduling lead to bottlenecks?
- Do dynamic load-balancing schemes require too much coordination to re-balance the load?
- What is the tradeoff of dynamic scheduling complexity versus performance improvement?
- Are there enough tasks to achieve high levels of concurrency? If not, processors may idle.
Types of Parallel Programs

- Flavors of parallelism
  - Data parallelism
    - all processors do same thing on different data
  - Task parallelism
    - processors are assigned tasks that do different things

- Parallel execution models
  - Data parallel
  - Pipelining (Producer-Consumer)
  - Task graph
  - Work pool
  - Master-Worker
Data Parallel

- Data is decomposed (mapped) onto processors
- Processors performance similar (identical) tasks on data
- Tasks are applied concurrently
- Load balance is obtained through data partitioning
  - Equal amounts of work assigned
- Certainly may have interactions between processors
- Data parallelism scalability
  - Degree of parallelism tends to increase with problem size
  - Makes data parallel algorithms more efficient
- Single Program Multiple Data (SPMD)
  - Convenient way to implement data parallel computation
  - More associated with distributed memory parallel execution
**Matrix - Vector Multiplication**

- $A \times b = y$
- Allocate tasks to rows of $A$
  \[ y[i] = \sum_j A[i,j] \times b[j] \]
- Dependencies?
- Speedup?
- Computing each element of $y$ can be done independently
Matrix-Vector Multiplication (Limited Tasks)

- Suppose we only have 4 tasks
- Dependencies?
- Speedup?

![Diagram of matrix-vector multiplication with tasks and dependencies]
Matrix Multiplication

- A x B = C
- A[i,:] • B[:,j] = C[i,j]

- Row partitioning
  - N tasks

- Block partitioning
  - N*N/B tasks

- Shading shows data sharing in B matrix
Granularity of Task and Data Decompositions

- Granularity can be with respect to tasks and data

- Task granularity
  - Equivalent to choosing the number of tasks
  - Fine-grained decomposition results in large # tasks
  - Large-grained decomposition has smaller # tasks
  - Translates to data granularity after # tasks chosen
    - consider matrix multiplication

- Data granularity
  - Think of in terms of amount of data needed in operation
  - Relative to data as a whole
  - Decomposition decisions based on input, output, input-output, or intermediate data
Mesh Allocation to Processors

- Mesh model of Lake Superior
- How to assign mesh elements to processors

- Distribute onto 8 processors
  - randomly
  - graph partitioning for minimum edge cut
Pipeline Model

- Stream of data operated on by succession of tasks
  - Task 1  Task 2  Task 3  Task 4
  - Tasks are assigned to processors

- Consider $N$ data units
- Sequential
  - 4 data units

- Parallel (each task assigned to a processor)
  - 8 data units

4-way parallel, but for longer time
Pipeline Performance

- $N$ data and $T$ tasks
- Each task takes unit time $t$
- Sequential time = $N*T*t$
- Parallel pipeline time = $\text{start} + \text{finish} + (N-2T)/T * t$
  
  \[ = O(N/T) \quad (\text{for } N \gg T) \]

- Try to find a lot of data to pipeline
- Try to divide computation in a lot of pipeline tasks
  - More tasks to do (longer pipelines)
  - Shorter tasks to do
- Pipeline computation is a special form of producer-consumer parallelism
  - Producer tasks output data input by consumer tasks
**Tasks Graphs**

- Computations in any parallel algorithms can be viewed as a task dependency graph.
- Task dependency graphs can be non-trivial:
  - Pipeline
  - Arbitrary (represents the algorithm dependencies)

Numbers are time taken to perform task.

(a) Task 1 → Task 2 → Task 3 → Task 4

(b) Task 1 → Task 2 → Task 3 → Task 4
Task Graph Performance

- Determined by the critical path (span)
  - Sequence of dependent tasks that takes the longest time

Min time = 27

○ Critical path length bounds parallel execution time

Min time = 34
**Task Assignment (Mapping) to Processors**

- Given a set of tasks and number of processors
- How to assign tasks to processors?
- Should take dependencies into account
- Task mapping will determine execution time

![Diagram](a) Total time = ?

![Diagram](b) Total time = ?
Task Graphs in Action

- Uintah task graph scheduler
  - C-SAFE: Center for Simulation of Accidental Fires and Explosions, University of Utah
  - Large granularity tasks

- PLASMA
  - DAG-based parallel linear algebra
  - DAGuE: A generic distributed DAG engine for HPC

Task graph for PDE solver
DAG of QR for a 4 × 4 tiles matrix on a 2 × 2 grid of processors.
Bag o’ Tasks Model and Worker Pool

- Set of tasks to be performed
- How do we schedule them?
  - Find independent tasks
  - Assign tasks to available processors
- Bag o’ Tasks approach
  - Tasks are stored in a bag waiting to run
  - If all dependencies are satisfied, it is moved to a ready to run queue
  - Scheduler assigns a task to a free processor
- Dynamic approach that is effective for load balancing
Master-Worker Parallelism

- One or more master processes generate work
- Masters allocate work to worker processes
- Workers idle if have nothing to do
- Workers are mostly stupid and must be told what to do
  - Execute independently
  - May need to synchronize, but must be told to do so
- Master may become the bottleneck if not careful
- What are the performance factors and expected performance behavior
  - Consider task granularity and asynchrony
  - How do they interact?
Master-Worker Execution Model (Li Li)

**M-W Execution Trace (Li Li)**

![Diagram showing M-W Execution Trace](image_url)
Search-Based (Exploratory) Decomposition

- 15-puzzle problem
- 15 tiles numbered 1 through 15 placed in 4x4 grid
  - Blank tile located somewhere in grid
  - Initial configuration is out of order
  - Find shortest sequence of moves to put in order

- Sequential search across space of solutions
  - May involve some heuristics
Parallelizing the 15-Puzzle Problem

- Enumerate move choices at each stage
- Assign to processors
- May do pruning
- Wasted work
Divide-and-Conquer Parallelism

- Break problem up in orderly manner into smaller, more manageable chunks and solve

- Quicksort example

```
5 12 11 1 10 6 8 3 7 4 9 2

1 3 4 2

1 2 3 4

1 2

3 4
```

```
5 12 11 10 6 8 7 9

5 12 11 10

5 6 8 7

5 6

5 6 7 8

9 10 12 11

11 12
```
Dense Matrix Algorithms

- Great deal of activity in algorithms and software for solving linear algebra problems
  - Solution of linear systems (\(Ax = b\))
  - Least-squares solution of over- or under-determined systems (\(\min ||Ax-b||\))
  - Computation of eigenvalues and eigenvectors (\(Ax=\lambda x\))
  - Driven by numerical problem solving in scientific computation

- Solutions involves various forms of matrix computations

- Focus on high-performance matrix algorithms
  - Key insight is to maximize computation to communication
Solving a System of Linear Equations

- $Ax=b$

$$
\begin{align*}
    a_{0,0}x_0 & + a_{0,1}x_1 & + \ldots & + a_{0,n-1}x_{n-1} & = b_0 \\
    a_{1,0}x_0 & + a_{1,1}x_1 & + \ldots & + a_{1,n-1}x_{n-1} & = b_1 \\
    \ldots & \\
    a_{n-1,0}x_0 & + a_{n-1,1}x_1 & + \ldots & + a_{n-1,n-1}x_{n-1} & = b_{n-1}
\end{align*}
$$

- Gaussian elimination (classic algorithm)
  - Forward elimination to $Ux=y$ ($U$ is upper triangular)
    - without or with partial pivoting
  - Back substitution to solve for $x$
  - Parallel algorithms based on partitioning of $A$
Sequential Gaussian Elimination

1. procedure GAUSSIAN ELIMINATION (A, b, y)
2. Begin
3.   for k := 0 to n - 1 do /* Outer loop */
4.     begin
5.       for j := k + 1 to n - 1 do
7.         y[k] := b[k]/A[k, k];
8.         A[k, k] := 1;
9.       for i := k + 1 to n - 1 do
10.      begin
11.         for j := k + 1 to n - 1 do
13.           b[i] := b[i] - A[i, k] x y[k];
15.       endfor; /*Line9*/
16.     endfor; /*Line3*/
17.   end GAUSSIAN ELIMINATION
### Computation Step in Gaussian Elimination

Consider the system of linear equations:

\[
\begin{align*}
5x + 3y &= 22 \\
8x + 2y &= 13
\end{align*}
\]

We solve for \( x \) and \( y \) using Gaussian elimination.

1. **Step 1:**
   - **Inactive part:**
   - **Active part:**
   - **Row \( k \):**
   - **Row \( i \):**

2. **Step 2:**
   - Update the value of \( A[k,j] \):
   - Update the value of \( A[i,j] \):

3. **Values:**
   - \( x = (22 - 3y) / 5 \)
   - \( y = (13 - 176/5) / (24/5 + 2) \)

By following these steps, we can solve the system of equations efficiently in parallel.
**Rowwise Partitioning on Eight Processes**

<table>
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<tr>
<th>P_0</th>
<th>1</th>
<th>(0.1)</th>
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<th>(0.3)</th>
<th>(0.4)</th>
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(a) Computation:

(i) \( A[k,j] := A[k,j]/A[k,k] \) for \( k < j < n \)

(ii) \( A[k,k] := 1 \)

(b) Communication:

One-to-all broadcast of row \( A[k,*] \)
Rowwise Partitioning on Eight Processes

(c) Computation:

for \( k < i < n \) and \( k < j < n \)

(ii) \( A[i,k] := 0 \) for \( k < i < n \)
2D Mesh Partitioning on 64 Processes

(a) Rowwise broadcast of $A[i,k]$ for $(k - 1) < i < n$

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(b) $A[k,j] := A[k,j]/A[k,k]$ for $k < j < n$

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(c) Columnwise broadcast of $A[k,j]$ for $k < j < n$

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Back Substitution to Find Solution

1. procedure BACK SUBSTITUTION (U, x, y)
2. begin
3. for k := n - 1 downto 0 do /* Main loop */
4. begin
5. x[k] := y[k];
6. for i := k - 1 downto 0 do
7. y[i] := y[i] - x[k] xU[i, k];
8. endfor;
9. end BACK SUBSTITUTION
Dense Linear Algebra (www.netlib.gov)

- Basic Linear Algebra Subroutines (BLAS)
  - Level 1 (vector-vector): vectorization
  - Level 2 (matrix-vector): vectorization, parallelization
  - Level 3 (matrix-matrix): parallelization
- LINPACK (Fortran)
  - Linear equations and linear least-squares
- EISPACK (Fortran)
  - Eigenvalues and eigenvectors for matrix classes
- LAPACK (Fortran, C) (LINPACK + EISPACK)
  - Use BLAS internally
- ScaLAPACK (Fortran, C, MPI) (scalable LAPACK)
Numerical Libraries

- **PETSc**
  - Data structures / routines for partial differential equations
  - MPI based
- **SuperLU**
  - Large sparse nonsymmetric linear systems
- **Hypre**
  - Large sparse linear systems
- **TAO**
  - Toolkit for Advanced Optimization
- **DOE ACTS**
  - Advanced CompuTational Software
Sorting Algorithms

- Task of arranging unordered collection into order
- Permutation of a sequence of elements
- Internal versus external sorting
  - External sorting uses auxiliary storage
- Comparison-based
  - Compare pairs of elements and exchange
  - $O(n \log n)$
- Noncomparison-based
  - Use known properties of elements
  - $O(n)$
Sorting on Parallel Computers

- Where are the elements stored?
  - Need to be distributed across processes
  - Sorted order will be with respect to process order

- How are comparisons performed?
  - One element per process
    - compare-exchange
    - interprocess communication will dominate execution time
  - More than one element per process
    - compare-split

- Sorting networks
  - Based on comparison network model

- Contrast with shared memory sorting algorithms
**Single vs. Multi Element Comparison**

- **One element per processor**
  
  - $a_i \rightarrow a_j$
  - $a_i, a_j$
  - $a_j, a_i$
  - $\min\{a_i, a_j\}$
  - $\max\{a_i, a_j\}$

- **Multiple elements per processor**
  
  - $1, 6, 8, 13 \rightarrow 2, 7, 9, 12$
  - $2, 7, 9, 12$
  - $1, 6, 8, 13$
  - $2, 7, 9, 12$
  - $1, 2, 6, 7, 8, 9, 10, 11, 12, 13$
  - $1, 2, 6, 7, 8, 9, 10, 11, 12, 13$
  - $1, 2, 6, 7, 8$
  - $9, 10, 11, 12, 13$
**Sorting Networks**

- Networks to sort $n$ elements in less than $O(n \log n)$
- Key component in network is a comparator
  - Increasing or decreasing comparator

![Comparator Diagrams](a)

- Comparators connected in parallel and permute elements
**Sorting Network Design**

- Multiple comparator stages (# stages, # comparators)
- Connected together by interconnection network
- Output of last stage is the sorted list
- \( O(\log_2 n) \) sorting time
- Convert any sorting network to sequential algorithm
Bitonic Sort

- Create a *bitonic sequence* then sort the sequence
- Bitonic sequence
  - sequence of elements \(<a_0, a_1, ..., a_{n-1}>\)
  - \(<a_0, a_1, ..., a_i>\) is monotonically increasing
  - \(<a_i, a_{i+1}, ..., a_{n-1}>\) is monotonically decreasing
- Sorting using *bitonic splits* is called *bitonic merge*
- **Bitonic merge network** is a network of comparators
  - Implement bitonic merge
- Bitonic sequence is formed from unordered sequence
  - Bitonic sort creates a bitonic sequence
  - Start with sequence of size two (default bitonic)
**Bitonic Sort Network**

Unordered sequence

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- ⊗ decrease
- ⊘ increase
Bitonic Merge Network

Bitonic sequence

Sorted sequence

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Parallel Bitonic Sort on a Hypercube

1. procedure BITONIC SORT(label, d)
2. begin
3. for i := 0 to d - 1 do
4. for j := i downto 0 do
5. if (i + 1)st bit of label = j th bit of label then
6. comp exchange max(j);
7. else
8. comp exchange min(j);
9. end BITONIC SORT
Parallel Bitonic Sort on a Hypercube

Step 1

Step 2

Last stage

Step 3

Step 4
Bubble Sort and Variants

- Can easily parallelize sorting algorithms of $O(n^2)$
- **Bubble sort** compares and exchanges adjacent elements
  - $O(n)$ each pass
  - $O(n)$ passes
  - Available parallelism?

- **Odd-even transposition sort**
  - Compares and exchanges odd and even pairs
  - After $n$ phases, elements are sorted
  - Available parallelism?
Odd-Even Transposition Sort

Unsorted

3  2  3  8  5  6  4  1

Phase 1 (odd)

2  3  3  8  5  6  1  4

Phase 2 (even)

2  3  3  5  8  1  6  4

Phase 3 (odd)

2  3  3  5  1  8  4  6

Phase 4 (even)

2  3  3  1  5  4  8  6

Phase 5 (odd)

2  3  1  3  4  5  6  8

Phase 6 (even)

2  1  3  3  4  5  6  8

Phase 7 (odd)

1  2  3  3  4  5  6  8

Phase 8 (even)

Sorted
Parallel Odd-Even Transposition Sort

1. procedure ODD-EVEN PAR(n)
2. begin
3.   id := process’s label
4.   for i := 1 to n do
5.     begin
6.       if i is odd then
7.         if id is odd then
8.           compare-exchange min(id + 1);
9.         else
10.        compare-exchange max(id - 1);
11.       if i is even then
12.         if id is even then
13.          compare-exchange min(id + 1);
14.        else
15.          compare-exchange max(id - 1);
16.     end for
17. end ODD-EVEN PAR
Quicksort

- Quick sort has average complexity of $O(n \log n)$
- Divide-and-conquer algorithm
  - Divide into subsequences where every element in first is less than or equal to every element in the second
  - Pivot is used to split the sequence
  - Conquer step recursively applies quicksort algorithm
- Available parallelism?
Sequential Quicksort

1. procedure QUICKSORT (A, q, r )
2. begin
3. if q < r then
4. begin
5. x := A[q];
6. s := q;
7. for i := q + 1 to r do
8. if A[i] ≤ x then
9. begin
10. s := s + 1;
11. swap(A[s], A[i ]);
12. end if
13. swap(A[q], A[s]);
14. QUICKSORT (A, q, s);
15. QUICKSORT (A, s + 1, r );
16. end if
17. end QUICKSORT
Parallel Shared Address Space Quicksort

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<tr>
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<th>P1</th>
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Parallel Shared Address Space Quicksort

Third Step

Fourth Step

Solution
Bucket Sort and Sample Sort

- **Bucket sort** is popular when elements (values) are uniformly distributed over an interval
  - Create $m$ buckets and place elements in appropriate bucket
  - $O(n \log(n/m))$
  - If $m=n$, can use value as index to achieve $O(n)$ time

- **Sample sort** is used when uniformly distributed assumption is not true
  - Distributed to $m$ buckets and sort each with quicksort
  - Draw sample of size $s$
  - Sort samples and choose $m-1$ elements to be *splitters*
  - Split into $m$ buckets and proceed with bucket sort
Parallel Sample Sort

Initial element distribution

Local sort & sample selection

Sample combining

Global splitter selection

Final element assignment
Graph Algorithms

- Graph theory important in computer science
- Many complex problems are graph problems

- $G = (V, E)$
  - $V$ finite set of points (vertices)
  - $E$ finite set of edges
  - $e \in E$ is a pair $(u, v)$, where $u, v \in V$
  - Unordered and ordered graphs
Graph Terminology

- Vertex *adjacency* if \((u,v)\) is an edge
- *Path* from \(u\) to \(v\) if there is an edge sequence starting at \(u\) and ending at \(v\)
- If there exists a path, \(v\) is *reachable* from \(u\)
- A graph is *connected* if all pairs of vertices are connected by a path
- A *weighted* graph associates weights with each edge
- *Adjacency matrix* is an \(n \times n\) array \(A\) such that
  - \(A_{i,j} = 1\) if \((v_i,v_j) \in E\); 0 otherwise
  - Can be modified for weighted graphs (\(\infty\) is no edge)
  - Can represent as *adjacency lists*
Graph Representations

- Adjacency matrix

- Adjacency list
Minimum Spanning Tree

- A spanning tree of an undirected graph $G$ is a subgraph of $G$ that is a tree containing all the vertices of $G$
- The minimum spanning tree (MST) for a weighted undirected graph is a spanning tree with minimum weight
- Prim’s algorithm can be used
  - Greedy algorithm
  - Selects an arbitrary starting vertex
  - Chooses new vertex guaranteed to be in MST
  - $O(n^2)$
  - Prim’s algorithm is iterative
Prim’s Minimum Spanning Tree Algorithm

1. procedure PRIM MST(V, E, w, r )
2. begin
3. VT := {r };
4. d[r ] := 0;
5. for all v ∈ (V - VT ) do
6. if edge (r, v) exists set d[v ] := w(r, v);
7. else set d[v ] := ∞;
8. while VT ≠ V do
9. begin
10. find a vertex u such that d[u ] := min{d[v]|v ∈ (V - VT )};
11. VT := VT ∪ {u };
12. for all v ∈ (V - VT ) do
13. d[v ] := min{d[v ],w(u, v)};
14. endwhile
15. end PRIM MST
Example: Prim’s MST Algorithm
Example: Prim’s MST Algorithm

(c) After the second edge has been selected

(d) Final minimum spanning tree

\( d[i] \)

\[
\begin{array}{ccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} \\
1 & 0 & 2 & 1 & 4 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
a & b & c & d & e & f \\
0 & 1 & 3 & \infty & \infty & 3 \\
b & 1 & 0 & 5 & 1 & \infty & \infty \\
c & 3 & 5 & 0 & 2 & 1 & \infty \\
d & \infty & 1 & 2 & 0 & 4 & \infty \\
e & \infty & \infty & 1 & 4 & 0 & 5 \\
f & 2 & \infty & \infty & \infty & \infty & 5 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
a & b & c & d & e & f \\
1 & 0 & 2 & 1 & 1 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
a & b & c & d & e & f \\
0 & 1 & 3 & \infty & \infty & 3 \\
b & 1 & 0 & 5 & 1 & \infty & \infty \\
c & 3 & 5 & 0 & 2 & 1 & \infty \\
d & \infty & 1 & 2 & 0 & 4 & \infty \\
e & \infty & \infty & 1 & 4 & 0 & 5 \\
f & 2 & \infty & \infty & \infty & \infty & 5 & 0 \\
\end{array}
\]
Parallel Formulation of Prim’s Algorithm

- Difficult to perform different iterations of the while loop in parallel because $d[v]$ may change each time.
- Can parallelize each iteration though.
- Partition vertices into $p$ subsets $V_i$, $i=0,...,p-1$.
- Each process $P_i$ computes
  \[ d_i[u] = \min \{ d_i[v] \mid v \in (V - V_T) \cap V_i \} \]
- Global minimum is obtained using all-to-one reduction.
- New vertex is added to $V_T$ and broadcast to all processes.
- New values of $d[v]$ are computed for local vertex.
- $O(n^2/p) + O(n \log p)$ (computation + communication)
Partitioning in Prim’s Algorithm

\[
\begin{array}{c|c|c}
\hline
\text{A} & \cdots & \cdots \\
\hline
\text{Processors} & 0 & 1 & \cdots & p-1 \\
\end{array}
\]

(a) $d[1..n]$
Single-Source Shortest Paths

- Find shortest path from a vertex \( v \) to all other vertices
- The shortest path in a weighted graph is the edge with the minimum weight
- Weights may represent time, cost, loss, or any other quantity that accumulates additively along a path
- Dijkstra’s algorithm finds shortest paths from vertex \( s \)
  - Similar to Prim’s MST algorithm
    - MST with vertex \( v \) as starting vertex
  - Incrementally finds shortest paths in greedy manner
  - Keep track of minimum cost to reach a vertex from \( s \)
  - \( O(n^2) \)
Dijkstra’s Single-Source Shortest Path

1. procedure DIJKSTRA SINGLE SOURCE SP(V, E, w, s)
2. begin
3. \( V_T := \{s\} \);
4. for all \( v \in (V - V_T) \) do
5. \( \text{if } (s, v) \text{ exists set } l[v] := w(s, v); \)
6. \( \text{else set } l[v] := \infty; \)
7. while \( V_T \neq V \) do
8. begin
9. find a vertex \( u \) such that \( l[u] := \min\{l[v] | v \in (V - V_T)\}; \)
10. \( V_T := V_T \cup \{u\}; \)
11. for all \( v \in (V - V_T) \) do
12. \( l[v] := \min\{l[v], l[u] + w(u, v)\}; \)
13. endwhile
14. end DIJKSTRA SINGLE SOURCE SP
Parallel Formulation of Dijkstra’s Algorithm

- Very similar to Prim’s MST parallel formulation
- Use 1D block mapping as before
- All processes perform computation and communication similar to that performed in Prim’s algorithm
- Parallel performance is the same
  - $O(n^2/p) + O(n \log p)$
  - Scalability
    - $O(n^2)$ is the sequential time
    - $O(n^2) / [O(n^2/p) + O(n \log p)]$
All Pairs Shortest Path

- Find the shortest path between all pairs of vertices
- Outcome is a $n \times n$ matrix $D=\{d_{i,j}\}$ such that $d_{i,j}$ is the cost of the shortest path from vertex $v_i$ to vertex $v_j$
- Dijsktra’s algorithm
  - Execute single-source algorithm on each process
  - $O(n^3)$
  - Source-partitioned formulation (use sequential algorithm)
  - Source-parallel formulation (use parallel algorithm)
- Floyd’s algorithm
  - Builds up distance matrix from the bottom up
Floyd’s All-Pairs Shortest Paths Algorithm

1. **procedure** FLOYD ALL PAIRS SP(A)
2. **begin**
3. \[ D^{(0)} = A; \]
4. **for** \( k := 1 \) **to** \( n \) **do**
5. **for** \( i := 1 \) **to** \( n \) **do**
6. **for** \( j := 1 \) **to** \( n \) **do**
7. \[ d^{(k)}_{i, j} := \min \ d^{(k-1)}_{i, j}, d^{(k-1)}_{i, k} + d^{(k-1)}_{k, j}; \]
8. **end** FLOYD ALL PAIRS SP
Parallel Floyd’s Algorithm

1. procedure FLOYD ALL PAIRS PARALLEL (A)
2. begin
3. \[ D^{(0)} = A; \]
4. for \( k := 1 \) to \( n \) do
5. forall \( P_{i,j} \), where \( i, j \leq n \), do in parallel
6. \[ d^{(k)}_{i,j} := \min d^{(k-1)}_{i,j}, d^{(k-1)}_{i,k} + d^{(k-1)}_{k,j}; \]
7. end FLOYD ALL PAIRS PARALLEL
Parallel Graph Algorithm Library – Boost

- Parallel Boost Graph Library
  - Andrew Lumsdaine, Indiana University
  - Generic C++ library for high-performance parallel and distributed graph computation
  - Builds on the Boost Graph Library (BGL)
    - offers similar data structures, algorithms, and syntax
  - Targets very large graphs (millions of nodes)
  - Distributed-memory parallel processing on clusters
Original BGL: Algorithms

- Searches (breadth-first, depth-first, A*)
- Single-source shortest paths (Dijkstra, Bellman-Ford, DAG)
- All-pairs shortest paths (Johnson, Floyd-Warshall)
- Minimum spanning tree (Kruskal, Prim)
- Components (connected, strongly connected, biconnected)
- Maximum cardinality matching
- Max-flow (Edmonds-Karp, push-relabel)
- Sparse matrix ordering (Cuthill-McKee, King, Sloan, minimum degree)
- Layout (Kamada-Kawai, Fruchterman-Reingold, Gursoy-Atun)
- Betweenness centrality
- PageRank
- Isomorphism
- Vertex coloring
- Transitive closure
- Dominator tree
Original BGL Summary

- Original BGL is large, stable, efficient
  - Lots of algorithms, graph types
  - Peer-reviewed code with many users, nightly regression testing, and so on
  - Performance comparable to FORTRAN.

- Who should use the BGL?
  - Programmers comfortable with C++
  - Users with graph sizes from tens of vertices to millions of vertices
Parallel BGL

- A version of C++ BGL for computational clusters
  - Distributed memory for huge graphs
  - Parallel processing for improved performance

- An active research project

- Closely related to the original BGL
  - Parallelizing BGL programs should be “easy”

A simple, directed graph…

distributed across 3 processors
Parallel Graph Algorithms

- Breadth-first search
- Eager Dijkstra’s single-source shortest paths
- Crauser et al. single-source shortest paths
- Depth-first search
- Minimum spanning tree (Boruvka, Dehne & Götz)
- Connected components
- Strongly connected components
- Biconnected components
- PageRank
- Graph coloring
- Fruchterman-Reingold layout
- Max-flow (Dinic’s)
Big-Data and Map-Reduce

- Big-data deals with processing large data sets
- Nature of data processing problem makes it amenable to parallelism
  - Looking for features in the data
  - Extracting certain characteristics
  - Analyzing properties with complex data mining algorithms
- Data size makes it opportunistic for partitioning into large # of sub-sets and processing these in parallel
- We need new algorithms, data structures, and programming models to deal with problems
A Simple Big-Data Problem

- Consider a large data collection of text documents
- Suppose we want to find how often a particular word occurs and determine a probability distribution for all word occurrences

Sequential algorithm

<table>
<thead>
<tr>
<th>Word</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>web</td>
<td>2</td>
</tr>
<tr>
<td>weed</td>
<td>1</td>
</tr>
<tr>
<td>green</td>
<td>2</td>
</tr>
<tr>
<td>sun</td>
<td>1</td>
</tr>
<tr>
<td>moon</td>
<td>1</td>
</tr>
<tr>
<td>land</td>
<td>1</td>
</tr>
<tr>
<td>part</td>
<td>1</td>
</tr>
</tbody>
</table>
Parallelization Approach

- **Map**: partition the data collection into subsets of documents and process each subset in parallel
- **Reduce**: assemble the partial frequency tables to derive final probability distribution

Parallel algorithm
Parallelization Approach

- **Map**: partition the data collection into subsets of documents and process each subset in parallel
- **Reduce**: assemble the partial frequency tables to derive final probability distribution

**Parallel algorithm**
Actually, it is not easy to parallel….

**Fundamental issues**
Scheduling, data distribution, synchronization, inter-process communication, robustness, fault tolerance, …

**Architectural issues**
Flynn’s taxonomy (SIMD, MIMD, etc.), network topology, bisection bandwidth, cache coherence, …

**Common problems**
Livelock, deadlock, data starvation, priority inversion, …dining philosophers, sleeping barbers, cigarette smokers, …

**Actually, Programmer’s Nightmare….”
Map-Reduce Parallel Programming

- Become an important distributed parallel programming paradigm for large-scale applications
  - Also applies to shared-memory parallelism
  - Becomes one of the core technologies powering big IT companies, like Google, IBM, Yahoo and Facebook.
- Framework runs on a cluster of machines and automatically partitions jobs into number of small tasks and processes them in parallel
- Can capture in combining Map and Reduce parallel patterns
Map-Reduce Example

MAP: Input data → <key, value> pair

Data Collection: split1

Data Collection: split 2

Data Collection: split n

Split the data to Supply multiple processors

CIS 431/531: Parallel Computing, University of Oregon
**MapReduce**

MAP: Input data ➔ <key, value> pair
REDUCE: <key, value> pair ➔ <result>

Split the data to
Supply multiple processors