1. In a weighted graph with start node $s$, there are often multiple shortest paths from $s$ to any other node. We want to use Dijkstra’s algorithm to count them (so assume no negative edge weights). To each node $v$, we add a field $v.numPaths$ initialized to 0 (with $s.numPaths$ initialized to 1). Modify the RELAX routine so that Dijkstra’s algorithm will determine both the length of the shortest path to all other nodes and the number of such shortest paths.
2. Using the graph below, show the start and finish times determined by DFS. Using those finish times, provide a topological sort of the nodes of the graph. As on the homework, visit nodes in alphabetical order.
3. Illustrate the Floyd-Warshall algorithm on the graph with the following weight matrix

\[
\begin{pmatrix}
0 & 8 & \infty & \infty \\
8 & 0 & 5 & 3 \\
\infty & 5 & 0 & 6 \\
\infty & 3 & 6 & 0 \\
\end{pmatrix}
\]

Show the intermediate matrices for \( k = 1, 2, 3, 4 \). Note that the graph is undirected (and hence the matrix is symmetric).
4. We need to place a sequence of billboards and are allowed to do so at certain specified locations along a road. Each location has a different cost, and we want to minimize the total cost of billboard placement. We cannot place none, since there is a penalty if the billboards are too far apart.

The billboards can be placed at mileposts $m_0, m_1, \ldots, m_n$ which have placement costs $c_0, c_1, \ldots, c_n$ respectively. We are required to place a billboard at locations $m_0$ and at $m_n$. The penalty we pay is $100$ for every integer multiple of 20 miles: for example, if a billboard is placed 17 miles after the previous one, there is no penalty; if a billboard is placed 42 miles after the previous one, we pay a penalty of $\lfloor \frac{42}{20} \rfloor \cdot 100 = 2 \cdot 100 = 200$ dollars.

To start a dynamic programming solution, we define subproblem $BP[i]$ to be the minimum cost of billboard placement for billboards $0, 1, 2, \ldots, i$, where a billboard is to be placed at locations $m_0$ and $m_i$.

(a) What is (are) the base case(s) for $BP$?

(b) Provide a recurrence relation for $BP$. 