1 Amortized analysis: The potential function method

We continue with the push and multi-pop example, but with the potential function method. As we have shown in the lecture as well as in Chapter 1.5, it suffices to find the right potential function $\Phi_i$ such that $\Phi_0 = 0$ and $\Phi_i \geq 0, \forall i$.

In our example, we choose $\Phi_i$ to be the number of items in stack after the $i$th operation. It is easy to verify that $\Phi_0 = 0$ and $\Phi_i \geq 0, \forall i \geq 1$.

With the definition of the potential function, one can easily derive the amortized complexity of each operation. Thus, the crucial part of the potential function method is to find an appropriate potential function.

- Push(): $t' = ti + \Phi_i - \Phi_{i-1} = 1 + 1 = 2$, which is $O(1)$.
  The change in the potential is an increase in one, which combines with the constant-time operations of push to yield a total amortized cost of 2.

- Multi-pop(): $t' = ti + \Phi_i - \Phi_{i-1} = k - k = 0$, which is $O(1)$.
  The potential decrease cancels the running time of multi-pop().