1 Asymptotic Growth Rate: big-O

Definition 1 (Big-O)  Given $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$, $f(n)$ is $O(g(n))$ if there exists a constant $c > 0$ and $n_0 \in \mathbb{N}$ such that $f(n) \leq cg(n)$, $\forall n \geq n_0$.

$f(n)$ is big-Oh of $g(n)$ (or, order $g(n)$). An abuse of the notation, $O(g(n))$ denotes the set of all functions that are big-Oh of $g(n)$, i.e.,

$$O(g(n)) = \{ f(n) : f(n) \text{ is big-Oh of } g(n) \}.$$

Example 1

$$20n^3 + 10n \log n + 5 \leq 20n^3 + 10n^3 + 5n^3 \quad \forall n \geq n_0 = 100$$

$$\leq 35n^3 \in O(n^3)$$

Remark: one can choose loose $c$ and $n_0$ from Definition 1. The key is then to prove inequalities and compare asymptotic behaviors between functions.