Lecture 11/16/16

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Reading: Chapter 3.3
Red-Black Trees

Bounded Depth Tree

- Trees with color nodes. Properties $\Rightarrow$ efficient updates.
- **Root Property**: the root is **black**.
- **External Property**: external node is **black**.
- **Internal Property**: the children of a red node are **black**.
- **Depth Property**: all the external nodes have the same **black depth**, which is defined as the number of black ancestors minus one.
Red-Black Trees
Correspondence between (2,4) Tree and Red-black Trees

Mappings

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- **(2,4) Tree $\Rightarrow$ Red-black Tree**: constructive (but) not unique mappings.
  - Root, external and internal property guaranteed by constructions.
  - (Black) depth property guaranteed by the depth property of (2,4) trees.