Lecture 11/07/16

Lecturer: Xiaodi Wu

Reading: Chapter 3.2
Re-balance it through the tri-node operation

Fix the $x, y, z$ in the previous slides

- Let $a, b, c$ be the inorder sequence of $x, y, z$.
- Let $T_0, T_1, T_2, T_3$ be the four sub-trees such that the inorder traversal is $T_0, a, T_1, b, T_2, c, T_3$.
- Change the tree to the following shape.
Cases: $y$ in the middle

Let $T_0, T_1, T_2, T_3$ be subtrees.

- $z, y, x$: full picture $T_0, z, T_1, y, T_2, x, T_3$ (in-order traversal).
Cases: $y$ in the middle after balance

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Similarly for $x, y, z$ case. This is called **single rotation**!
Correctness

Claims

- Rotation design $\Rightarrow$ binary search tree.
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Analyze possible cases of unbalanced nodes

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- Remove $w$ could decrease the height of some subtree. However, with at most 1 unbalanced node ($z$). Why?
Unbalance Nodes: removal!

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- Let $y$ be $z$’s child with higher height. Let $x$ be $y$’s child with higher height (could be a tie).
- Claim: $x, y$ are not $w$’s ancestor. Why?
Cont’d: Removal

Implementation & Complexity

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▶ The height of $b$, however, might decrease by one. Thus need to repeat the procedure until reach the root.
Cont’d: Removal

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