AVL tree: Update

Updates like BST

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Unbalance Nodes: insertion!

Analyze possible cases of unbalanced nodes

- Let $w$ be the inserted node. Follow the path from $w$ to the root (update height). How?
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- Understand how heights are updated. Insertion can only increase the height of some subtree.
- Could be more unbalanced nodes. $z$ is the first. Why?
Re-balance it through the tri-node operation

Fix the $x, y, z$ in the previous slides

- Let $a, b, c$ be the inorder sequence of $x, y, z$. 
Re-balance it through the tri-node operation

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- Let $a, b, c$ be the inorder sequence of $x, y, z$.
- Let $T_0, T_1, T_2, T_3$ be the four sub-trees such that the inorder traversal is $T_0, a, T_1, b, T_2, c, T_3$. 
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- Change the tree to the following shape.
Tri-node operation based on \( x, y, z \)

Properties of \( x, y, z \)

- \( z \): the first unbalanced node. Let \( y \) be \( z \)'s child with higher height. Let \( x \) be \( y \)'s child with higher height (could be a tie).
Tri-node operation based on $x, y, z$

Properties of $x, y, z$

- $z$: the first unbalanced node. Let $y$ be $z$’s child with higher height. Let $x$ be $y$’s child with higher height (could be a tie).
- Then there are four possible relative relations of $x, y, z$ in the in-order traversal. Why not 6?
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- \(z\): the first unbalanced node. Let \(y\) be \(z\)'s child with higher height. Let \(x\) be \(y\)'s child with higher height (could be a tie).
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  - $z, y, x$.
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  - $y, x, z$. 
Cases: $y$ in the middle

Let $T_0, T_1, T_2, T_3$ be subtrees.

- $z, y, x$: full picture $T_0, z, T_1, y, T_2, x, T_3$ (in-order traversal).
Cases: $y$ in the middle

Let $T_0, T_1, T_2, T_3$ be subtrees.

- $z, y, x$: full picture $T_0, z, T_1, y, T_2, x, T_3$ (in-order traversal).
Cases: $y$ in the middle after balance

Let $T_0, T_1, T_2, T_3$ be subtrees.

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Similarly for $x, y, z$ case. This is called **single rotation**!
Cases: $x$ in the middle

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Similarly for $y, x, z$ case. This is called **double rotation**!
Find the unbalanced $z$ if any and the $y, x$.

$O(h) = O(\log(n))$.

In total, $O(\log(n))$. 

Only need to perform once. Update the heights.
Implementation & Complexity

- Find the unbalanced $z$ if any and the $y, x$. $O(h) = O(\log(n))$. 
Find the unbalanced $z$ if any and the $y, x$. $O(h) = O(\log(n))$. 
Determine the in-order relationship of $z, y, x$. 
Implementation & Complexity

- Find the unbalanced $z$ if any and the $y, x$. \( O(h) = O(\log(n)) \).
- Determine the in-order relationship of $z, y, x$. \( O(1) \)
Implementation & Complexity

- Find the unbalanced $z$ if any and the $y, x$. $O(h) = O(\log(n))$.
- Determine the in-order relationship of $z, y, x$. $O(1)$
- Perform tri-node operation.
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- Find the unbalanced \( z \) if any and the \( y, x \). \( O(h) = O(\log(n)) \).
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