the power of \( \exists \)
non-deterministic polynomial

• non-determinism allows a choice of next step
• used for yes/no problems
• if there is a choice of next steps that makes it say yes, then it “accepts”
• runs in polynomial time

• ACCEPTS iff \( \exists \) poly-length computation leading to “yes”
some of our fave NP problems

3SAT: given 3cnf formula F on variables $x_1, x_2, \ldots, x_n$, is there an assignment of true/false to the $x_i$ which makes $F[x_1, \ldots, x_n]$ true?

ex: $(x_3 \lor \neg x_5 \lor \neg x_2) \land (x_1 \lor x_2 \lor \neg x_3) \land (x_3 \lor \neg x_5 \lor \neg x_2) \land (x_3 \lor \neg x_5 \lor \neg x_2)$

3cnf instance:
- a literal is an $x_i$ or an $\neg x_i$
- a clause is the OR of up to 3 literals
- a formula is the AND (conjunction) of clauses

nondeterministic algorithm
for $i = 1$ to $n$
  set $x[i] = 0$ or $1$ (nondet step)
if $F[x[1], \ldots, x[n]]$ is true
  then ACCEPT
else REJECT

if F is satisfiable, there exists a way for this algorithm to reach an accepting state
**3COL:** Given a graph G, is there an assignment of 3 colors to the nodes of G so that no two adjacent nodes have the same color?

**HP:** (Hamilton path) Does G have a path that starts at one node, ends at another, and visits all the other nodes exactly once?

**TSP:** (travelling salesman problem) Given a weighted graph G and a bound B, is there a cycle that traverses all nodes of G and has total weight at most B?

**LP:** Given a graph G and integer B, is there a simple path in G of length at least B?
many characterizations of NP

- problems with short proofs that can be verified in poly-time
- a set $A$ is in $NP$ if there is a poly-time checkable relation $R$ such that
  \[ A = \{ x \mid \exists y \ (|y| \leq |x|^k) \ R(x,y) \} \]
- existential second-order logic (Fagin’s Theorem)
NP-Complete

the hardest problems in NP

if one NP-complete problem can be solved in poly-time, then all of NP can be solved in poly-time

main point:
no one knows if any NP complete problem can be efficiently solved – it is one of the big open problems of computer science and/or mathematics
handled by reductions

for example, the problem of 3SAT can be reduced to 3COL, which we write $3\text{SAT} \leq_p 3\text{COL}$

to be done in class, and is a “proof by widget”, but uses the graph below

**note**: this graph cannot be colored with three colors if the corners have the same color