1. [15 pts] Consider the following factor graph and tables. Assuming that the initial messages from variables to factors are uniform (e.g., \((0.5, 0.5)\)), how many iterations will loopy BP take to converge? What are the resulting marginals? Are they correct? Explain your reasoning and show your work. (HINT: You don’t need to manually compute every message separately.)

2. K&F 9.3. Please answer for two specific types of queries:

(a) Probability of evidence \(P(E = e)\). (Note that \(P(E = e)\) in a BN can be computed by reducing the BN with evidence \(E = e\) and summing out all remaining variables \(X - E\). As discussed in class, this works because reducing the BN with evidence is equivalent to “crossing out” the entries in the probability distribution that don’t match the evidence, so the sum of the remaining table entries is precisely the probability of that evidence.)

(b) Conditional probabilities \(P(Y | E = e)\).
3. [Grads only] Consider a Markov network consisting of a single loop of pairwise potentials:

\[ P(X_1, \ldots, X_n) = \frac{1}{Z} \phi_1(X_1, X_2)\phi_2(X_2, X_3) \ldots \phi_{n-1}(X_{n-1}, X_n)\phi_n(X_n, X_1) \]

(a) Show that eliminating any variable \( X_i \) will yield an intermediate factor with a scope of three variables.

(b) Use your result from part a to prove that eliminating all variables from any network with cycles is \( \Omega(|Val(X_i)|^3) \), where \( |Val(X_i)| \) is the minimum number of values of any variable \( X_i \in X \). (Recall that Big-\( \Omega \) is analogous to Big-O, but a lower bound instead of an upper bound.)