CIS 441/541: Introduction to Computer Graphics
Lecture 3: Interpolation
Where we are...

- We haven’t talked about how to get triangles into position.
  - Arbitrary camera positions through linear algebra
- We haven’t talked about shading
- On Friday, we tackled this problem:
  How to deposit triangle colors onto an image?

Still don’t know how to:
1) Vary colors (easy)
2) Deal with triangles that overlap

Today’s lecture will go over the key operation to do these two. Friday’s lecture will tell us how to do it.
Outline

- Project 1B
- Scanline review
- Interpolation along a triangle
- Project 1C
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- Project 1B
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Project 1B: Questions?

- Clear what to do?
- Clear what to hand in?
- Follow the prompts as described ... the creative part of this class comes at the end.
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There are many ways we can select color for a pixel.
Problem: how to deposit triangle colors onto an image?

- Let’s take an example:
  - 12x12 image
  - Red triangle
    - Vertex 1: (2.5, 1.5)
    - Vertex 2: (2.5, 10.5)
    - Vertex 3: (10.5, 1.5)
  - Vertex coordinates are with respect to pixel locations
Our desired output

How do we make this output? Efficiently?
Don’t need to consider any pixels outside these lines.
Scanline algorithm: consider all rows that can possibly overlap.

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Scanline algorithm: consider all rows that can possibly overlap.

We will extract a “scanline”, i.e. calculate the intersections for one row of pixels.

Don’t need to consider any Pixels outside these.
- Red triangle
  - Vertex 1: (2.5, 1.5)
  - Vertex 2: (2.5, 10.5)
  - Vertex 3: (10.5, 1.5)
- Red triangle
  - Vertex 1: (2.5, 1.5)
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  - Vertex 2: (2.5, 10.5)
  - Vertex 3: (10.5, 1.5)

What are the end points?
Red triangle
- Vertex 1: (2.5, 1.5)
- Vertex 2: (2.5, 10.5)
- Vertex 3: (10.5, 1.5)

What are the end points?
(2.5, 5)
- Red triangle
  - Vertex 1: (2.5, 1.5)
  - Vertex 2: (2.5, 10.5)
  - Vertex 3: (10.5, 1.5)

What are the end points?
Red triangle
- Vertex 1: (2.5, 1.5)
- Vertex 2: (2.5, 10.5)
- Vertex 3: (10.5, 1.5)

\[ Y = mx + b \]

\[ 10.5 = m \times 2.5 + b \]
\[ 1.5 = m \times 10.5 + b \]

\[ \Rightarrow 9 = -8m \]
\[ m = -1.125 \]
\[ b = 13.3125 \]
\[ 5 = -1.125 \times x + 13.3125 \]
\[ x = 7.3888 \]

What are the end points?
(2.5, 5)
Scanline algorithm: consider all rows that can possibly overlap.

Don’t need to consider any Pixels outside these lines.

2.5

X

X

X

X

7.3888

Y=5
Scanline algorithm: consider all rows that can possibly overlap.

Don't need to consider any pixels outside these lines.

Color is deposited at (3,5), (4,5), (5,5), (6,5), (7,5)
Determine rows of pixels triangles can possibly intersect

- Call them rowMin to rowMax
  - rowMin: ceiling of smallest Y value
  - rowMax: floor of biggest Y value

For r in [rowMin → rowMax] ; do
  - Find end points of r intersected with triangle
    - Call them leftEnd and rightEnd
  - For c in [ceiling(leftEnd) → floor(rightEnd)] ; do
    - ImageColor(r, c) ← triangle color
Scanline algorithm

- Determine rows of pixels triangles can possibly intersect

- For \( r \) in \([\text{rowMin} \rightarrow \text{rowMax}]\) ; do
  - Find end points of \( r \) intersected with triangle
    - Call them \( \text{leftEnd} \) and \( \text{rightEnd} \)
  - For \( c \) in \([\text{ceiling}(\text{leftEnd}) \rightarrow \text{floor}(\text{rightEnd})]\) ; do
    - \( \text{ImageColor}(r, c) \leftarrow \text{triangle color} \)

For \( r = 5 \), we call \( \text{ImageColor} \) with \((5,3), (5,4), (5,5), (5,6), (5,7)\)

Y values from 1.5 to 10.5 mean rows 2 through 10

For \( r = 5 \), \( \text{leftEnd} = 2.5 \), \( \text{rightEnd} = 7.3888 \)
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What is a field?

Example field (2D): temperature over the United States
How much data is needed to make this picture?

Example field (2D): temperature over the United States
Linear Interpolation for Scalar Field $F$
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- General equation to interpolate:
  \[ F(X) = F(A) + t \cdot (F(B) - F(A)) \]
- $t$ is proportion of $X$ between $A$ and $B$
  \[ t = \frac{(X-A)}{(B-A)} \]
Quiz Time #4

- \( F(3) = 5, \ F(6) = 11 \)
- What is \( F(4) \)? \( = 5 + \frac{(4-3)}{(6-3)}(11-5) = 7 \)

- General equation to interpolate:
  - \( F(X) = F(A) + t*(F(B)-F(A)) \)
  - \( t \) is proportion of \( X \) between \( A \) and \( B \)
  - \( t = \frac{(X-A)}{(B-A)} \)
Consider a single scalar field defined on a triangle.
Consider a single scalar field defined on a triangle.

- \( V_1 \): \( F(V_1) = 10 \)
- \( V_2 \): \( F(V_2) = 2 \)
- \( V_3 \): \( F(V_3) = -2 \)
What is $F(V4)$?

- $F(V1) = 10$
- $F(V2) = 2$
- $F(V3) = -2$

Point $V4$, at $(0.5, 0.25)$
What is F(V4)?

F(V1) = 10
F(V2) = 2
F(V3) = -2
Steps to follow:

- Calculate V5, the left intercept for Y=0.25
- Calculate V6, the right intercept for Y=0.25
- Calculate V4, which is between V5 and V6
What is the X-location of V5?

F(V1) = 10

F(V2) = 2

F(v1) = A  \rightarrow  F(0) = 0
F(v2) = B  \rightarrow  F(1) = 1
F(v) = A + ((v-v1)/(v2-v1))*(B-A):

F(v) = 0.25, find v

0.25 = 0 + ((v-0)/(1-0))*(1-0)

v = 0.25
What is the F-value of V5?

F(V1) = A \quad \rightarrow \quad F(0) = 10
F(V2) = B \quad \rightarrow \quad F(1) = 2
F(v) = A + \frac{(v-v_1)/(v_2-v_1)}{(B-A)}

v = 0.25, find F(v)

F(v) = 10 + \frac{(0.25-0)/(1-0)}{(2-10)}
= 10 + 0.25*(-8) = 10 - 2 = 8
What is the X-location of V6?

Y-axis

Y=1

Y=0

Y=0.5

F(V2) = 2

X-axis

X=0

X=0.5

X=1

X=1.5

X=2

F(V1) = 10

F(V3) = -2

F(v1) = A

F(v2) = B

F(v) = A + ((v-v1)/(v2-v1))*(B-A):

F(v) = 0.25, find v

0.25 = 1 + ((v-1)/(2-1))*(0-1)

= 1 + (v-1)*(-1)

0.25 = 2 - v

v = 1.75
What is the F-value of V6?

F(v1) = A \rightarrow F(1) = 2
F(v2) = B \rightarrow F(2) = -2
F(v) = A + ((v-v1)/(v2-v1))*(B-A);

v = 1.75, find F(v)

F(v) = 2 + ((1.75-1)/(2-1)*(-2 - +2)
= 2 + (.75)*(-4)
= 2 - 3
= -1
What is the F-value of V5?

L(V5) = (0.25, 0.25)
F(V5) = 8

L(V6) = (1.75, 0.25)
F(V6) = -1
What is the F-value of V5?

F(v) = A + ((v-v1)/(v2-v1))*(B-A):

v = 0.5, find F(v)

F(v) = 8 + ((0.5-0.25)/(1.75-0.25))*(-1-8)
= 8 + (0.25/1.5)*9 = 8-1.5 = 6.5
Visualization of $F$

How do you think this picture was made?
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Arbitrary Triangles

How do we handle arbitrary triangles?

- But the implementation for these three triangles vary:
Arbitrary Triangles

How do we handle arbitrary triangles?

- But the implementation for these three triangles vary:

Solve for location of this point and then solve two “base cases”.
Arbitrary Triangles

- Project #1B: implement the scanline algorithm for triangles with “flat bottoms”
- Project #1C: arbitrary triangles
Goal: apply the scanline algorithm to arbitrary triangles and output an image.

Extend your project1B code

File proj1c_geometry.vtk available on web (80MB)

File “reader.cxx” has code to read triangles from file.

No Cmake, project1c.cxx