**Estimation of Relative Performance**

- Output comparison between two or more alternative systems
- Common Random Numbers (CRN)
- Comparison of several systems
- Optimization via simulation
Comparing Alternatives

• Usually, simulation is used for more than just a single model “configuration”
• Often want to compare alternatives, select or search for the best (via some criterion)
• Simple processing system: What would happen if the arrival rate were to double?
  - Cut interarrival times in half
  - Rerun the model for double-time arrivals
  - Make five replications

Comparing Two Alternatives

• General Idea: Approach I
  - Select some primary performance measure for comparison (e.g. production rate, average time in system, server utilization, etc…), denoted as $\Theta_i$ for alternative $i$.
  - Run alternative $i$ for $R_i$ replications to find the mean, standard deviation and confidence interval for performance parameter $\Theta_i$.
  - If the CI’s of each alternative overlap, you cannot conclude there is statistical difference between the alternatives.
  - What if you increased the number of replications?
  - What if you changed the significance level ($\alpha$) of the CI?
Comparing Two Alternatives

• **General Idea: Approach II**
  - Select some primary performance measure for comparison (e.g. production rate, average time in system, server utilization, etc...), denoted as $\theta_i$ for alternative $i$.
  - Run alternative $i$ for $R_i$ replications to obtain the performance parameter $\theta_{ri}$, where $r$ refers to the replication.
  - Find the difference between the performance measure for each replication ($\theta_{r1} - \theta_{r2}$).
  - Compute the mean, standard deviation and CI for ($\theta_{r1} - \theta_{r2}$).
  - If the CI of ($\theta_{r1} - \theta_{r2}$) contains the value 0, then there is no statistical difference between the two alternatives.

Common Random Numbers (CRN)

• Step 4 of Lab 11 was possible because the same random number stream was used to compare each alternative. (The only change between both systems was the arrival rate).
• This technique is referred to as Common Random Numbers (CRN).
• Usually possible only when changing parameter values between alternatives. If logic changes, or the execution of random numbers, then this technique is not valid.
Common Random Numbers (CRN)

- For CRN case:
  \[
  \left(\bar{Y}_1 - \bar{Y}_2\right) \pm t_{\alpha/2, v} s.e(\bar{Y}_1 - \bar{Y}_2)
  \]
  where
  \[s.e(\bar{Y}_1 - \bar{Y}_2) = \frac{S_D}{\sqrt{R}}, \quad \text{if } D_r = Y_{i1} - Y_{i2}\]

  for differences \[\bar{D} = \bar{Y}_{i1} - \bar{Y}_{i2}\] and \[S_D^2 = \frac{1}{R-1} \sum_{r=1}^{R} (D_r - \bar{D})\]
  and \[v = R - 1\]

Comparison of Several System Designs

- How do you compare multiple system designs?
- Depends on the goal:
  1. Estimation of the parameter, \(\Theta_i\) for alternative \(i\).
  2. Comparison of \(\Theta_i\) to some control \(\Theta_1\) which might be an existing system.
  3. All pairwise comparisons \(\Theta_i - \Theta_j, i \neq j\)
  4. Selection of the best \(\Theta_i\).
- Goals 1, 2, 3 all use CI’s. Goal 4 requires a type of optimization approach.
- Performing analysis on multiple alternatives is somewhat laborious, but some simulation packages facilitate this process.
Optimization via Simulation

• Goal 4 was to find the best $\Theta_i$.
• What if there are many (thousands or millions) of alternatives to choose from?
• One approach is to use meta-heuristics.
  1. Take an Evolutionary Computation course to learn more about meta-heuristics.
  2. Some software packages have optimization modules.