The objective of this homework is to understand the basic probability distributions used in computer simulation of discrete-event systems. Upon completion of the assignment, the student must understand the different probability distributions, their parameters, and their main characteristics.

1. An industrial chemical that will retard the spread of fire in paint has been developed. The local sales representative has estimated, from past experience, that 48% of the sales calls will result in an order.
   
   (a) What is the probability that the first order will come on the fourth sales call of the day?
   (b) If eight sales calls are made in a day, what is the probability of receiving exactly six orders?
   (c) If four sales calls are made before lunch, what is the probability that one or fewer results in an order?

2. The number of hurricanes hitting the coast of Florida annually has a Poisson distribution with a mean of 0.8.
   
   (a) What is the probability that more than two hurricanes will hit the Florida coast in a year?
   (b) What is the probability that exactly one hurricane will hit the coast of Florida in a year?

3. Records indicate that 1.8% of the entering students at a large state university drop out of school by midterm. What is the probability that three or fewer students will drop out of a random group of 200 entering students?

4. Lane Braintwain is quite a popular student. Lane receives, on the average, four phone calls a night (Poisson-distributed). What is the probability that, tomorrow night, the number of calls received will exceed the average by more than one standard deviation?

5. A random variable \( X \) that has pmf given by \( p(x) = \frac{1}{(n+1)} \) over the range \( R_X = \{0, 1, 2, \ldots, n\} \) is said to have a discrete uniform distribution.
   
   (a) Find the mean and variance of this distribution. \textit{Hint:}
   \[
   \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}
   
   (b) If \( R_X = \{a, a+1, a+2, \ldots, b\} \), compute the mean and variance of \( X \).
6. The lifetime, in years, of a satellite placed in orbit is given by the following pdf:

\[ f(x) = \begin{cases} 
0.4e^{-0.4x}, & x \geq 0 \\
0, & \text{otherwise} 
\end{cases} \]

(a) What is the probability that this satellite is still "alive" after 5 years?
(b) What is the probability that the satellite dies between 3 and 6 years from the time it is placed in orbit?

For graduate students:

7. Show that the geometric distribution is memoryless.

8. An aircraft has dual hydraulic systems. The aircraft switches to the standby system automatically if the first system fails. If both systems fail, the plane will crash. Assume that the life of a hydraulic system is exponentially distributed, with a mean of 2000 air hours.

(a) If the hydraulic systems are inspected every 2500 hours, what is the probability that an aircraft will crash before that time?
(b) What danger would there be in moving the inspection point to 3000 hours?

9. Three shafts are made and assembled into a linkage. The length of each shaft, in centimeters, is distributed as follows:

- Shaft 1: \( N(60, 0.09) \)
- Shaft 2: \( N(40, 0.05) \)
- Shaft 3: \( N(50, 0.11) \)

(a) What is the distribution of the length of the linkage?
(b) What is the probability that the linkage will be longer than 150.2 centimeters?
(c) The tolerance limits for the assembly are (149.83, 150.21). What proportion of assemblies are within the tolerance limits? [Hint: If \( \{X_i\} \) are \( n \) independent normal random variables, and if \( X_i \) has mean \( \mu_i \) and variance \( \sigma_i^2 \), then the sum

\[ Y = X_1 + X_2 + \cdots + X_n \]

is normal with mean \( \sum_{i=1}^{n} \mu_i \) and variance \( \sum_{i=1}^{n} \sigma_i^2 \).]