Matrix Multiplication and Graph Search

CIS 315
Standard $O(n^3)$ matrix multiplication

**input:** $n \times n$ matrices $A$ and $B$ (of int)

**output:** product $C = A \times B$

for $i=1$ to $n$
  for $j=1$ to $n$
    $C[i,j] = 0$
    for $k=1$ to $n$
      $C[i,j] = C[i,j] + A[i,k] \times B[k,j]$
Matrix multiplication over \( \{0,1\} \)

**Input:** \( n \times n \) matrices \( A \) and \( B \) (of boolean)

**Output:** product \( C = A* B \)

\[
\begin{align*}
\text{for } i &= 1 \text{ to } n \\
\quad \text{for } j &= 1 \text{ to } n \\
\quad \quad C[i,j] &= \text{false} \\
\quad \quad \text{for } k &= 1 \text{ to } n \\
\quad \quad \quad C[i,j] &= C[i,j] \lor (A[i,k] \land B[k,j])
\end{align*}
\]

+ becomes OR (\( \lor \)) and * becomes AND (\( \land \))
Transitive closure

- $M$ is the adjacency matrix
- $M^2$ (using boolean matrix mult) tells us about paths of length 2
- ... and $M^k$ about paths of length $k$
- the only $k$ that matter are $0 \leq k < V$
- $M^* = M^0 + M^1 + M^2 + ... + M^{V-1}$
- $M^* = (I + M)^V$
for $i=1$ to $n$
  for $j=1$ to $n$

  $W^{\leq 2}[i,j] = \begin{cases} 0 & \text{if } i=j \\ \infty & \text{else} \end{cases}$

for $k=1$ to $n$

  $W^{\leq 2}[i,j] = \text{MIN}(W^{\leq 2}[i,j], W[i,k]+W[k,j])$
BFS(G,s)
1 for each vertex u in V-{s}
2 u.color = WHITE
3 u.dist = infinity
4 u.prev = nil
5 s.color = GRAY
6 s.dist = 0
7 s.prev = nil
8 Q = empty
9 ENQUEUE(Q,s)
10 while Q not empty
11 u = DEQUEUE(Q)
12 for each v in ADJ(u) -- adjacency list of u
13 if v.color = WHITE
14 v.color = GRAY
15 v.dist = u.dist + 1
16 v.prev = u
17 ENQUEUE(Q,v)
18 u.color = BLACK
Depth-First Search (page 604 text)

DFS(G)
1 for each vertex u in V
2 u.color = WHITE
3 u.prev = nil
4 time = 0
5 for each vertex u in V
6 if u.color = WHITE
7 DFS-Visit(G,u)

DFS-Visit(G,u)
1 time = time + 1
2 u.disc = time
3 u.color = GRAY
4 for each v in adjacency list of u
5 if v.color = WHITE
6 v.prev = u
7 DFS-Visit(G,v)
8 u.color = BLACK
9 time = time +1
10 u.finish = time

white - not seen yet
gray - in process
black - done