1. Provide solutions (using big-Oh or big-Theta) for the following recurrence relations.
   (a) \( t(n) = 7t\left(\frac{n}{5}\right) + n^2 \)
   (b) \( t(n) = 7t\left(\frac{n}{5}\right) + 313n \)
   (c) \( t(n) = 36t\left(\frac{n}{6}\right) + n^2 \)
   (d) \( t(n) = 27t\left(\frac{n}{3}\right) + n(\log n)^3 \)
   [8 points]

2. Into an initially empty AVL tree, insert the following values:
   1, 3, 25, 20, 35, 15, 12, 18, 5, 10, 29.
   [11 points]

3. Insert the values above into an initially empty 2-3-4 tree. [10 points]

4. What are the run-times of the following pieces of code?
   (a) for \( i = 1 \) to \( n*(\log n) \)
       for \( j = 1 \) to \( i \)
       \( \text{sum}++ \)
   (b) for \( i = 1 \) to \( n \)
       \( j = i \)
       while (\( j > 1 \)) {
         \( \text{sum} ++ \)
         \( j = j \div 2 \)
       }
   [8 points]

TURN THE PAGE OVER FOR QUESTION 5
5. Suppose we augment a BST by adding to each node a field called \textit{size}. So given a node \( p \),
the field \( p.size \) contains the number of nodes in the subtree of \( p \).

We want to use this field to efficiently implement a \textit{select} method. The idea is that \textit{select}(k,p)
will find the \( k \)th smallest value in the subtree of \( p \) for each integer \( k \) with \( 1 \leq k \leq p.size \). Thus, \textit{select}(1, T.root) returns the smallest value in \( T \), \textit{select}(2, T.root) returns the 2nd smallest value stored in \( T \), and \textit{select}(T.root.size, T.root) returns the largest value in \( T \).

Imagine a tree \( T \) with the following characteristics at the top (let \( r = T.root \) be the root pointer):

\begin{itemize}
  \item \( r.size = 22 \) (the tree contains 22 values)
  \item \( r.left.size = 15 \) (the left subtree contains 15 values)
  \item \( r.right.size = 6 \) (the right subtree contains 6 values)
\end{itemize}

(a) If we execute \textit{select}(5, r), will the result be found in the left or right subtree of \( r \)? How about \textit{select}(18, r)? Or \textit{select}(16, r)?

(b) A call to \textit{select}(22, r) will find the result in the right subtree of \( r \). For what value of \( k \) will \textit{select}(k, r.right) return the same thing?

(c) Write a short procedure (maybe recursive) to find \textit{select}(k, p) whose run-time is bounded by the height of the tree.

[13 points]

Total: 50 points