1. Exercise 12.2-4, p 293 [4 points]

2. In class we defined the internal path length $I$ and the external path length $E$, both measures of a binary tree. If that tree has $n$ (internal) nodes, show that $E = I + 2n$. (This is exercise B.5-5, p 1180.) This should be a careful proof, most likely by induction. [7 points]

3. Consider the tree of figure 1. How many different permutations of the values 1 through 10, when inserted in that order, will yield this particular tree? [6 points]

![Figure 1: The BST for problem 3.](image)

4. How many permutations of $1, 2, \ldots, n$ yield a skew tree? (Since any one skew tree is generated by just one permutation, this question is asking for the number of skew trees of $n$ nodes.) Explain your formula. [5 points]

5. The first sentence of exercise 12.4-2, p 303. [4 points]

6. Insert into an initially empty AVL tree the following values:

   8, 12, 15, 5, 6, 2, 20, 21, 14.

   [7 points]
7. Suppose that an AVL implementation stores the height of the subtree in each node to calculate the balance factors. Also suppose that 4 bits (unsigned) are allocated for this purpose. What is the fewest number of nodes in an AVL tree that could cause one of these fields to overflow? What is the largest number of nodes that could be accommodated without overflow? (Use the $g_k$, related to the Fibonacci numbers, from class.) [6 points]

Total: 41 points

Notes:

• (Q2) We had $I = \sum_{v \in V} d(v)$, where $V$ is the set of nodes and $d(v)$ is the depth of a node. $E$ is defined similarly, over all external nodes. You will want to use induction.

• (Q3) Consider a tree where
  
  – the left subtree contains $n$ nodes and is generated by $r$ permutations
  – the right subtree contains $m$ nodes and is generated by $s$ permutations

  Then the whole tree contains $n + m + 1$ nodes and is generated by $r \cdot s \cdot \binom{n + m}{n}$ permutations.

• (Q7) With 4 bits, the value $2^4 - 1 = 15$ is the largest value that will fit. 16 would cause overflow.