CSE 413/513: Advanced Data Structures

Homework #2
Due in class on Wednesday, October 12, 2011

Guidelines: You may brainstorm with others, but please write up the answers by yourself. Acknowledge all collaborations and external resources used.

1. Recall that adding an item to a d-ary heap takes $O(\log_d n)$ time and removing the minimum takes $O(d \log_d n)$ time, where $n$ is the total number of items in the heap. If there are $n^2$ insert operations and $n$ delete-min operations, the total time taken for all operations is:

$$O(n^2 \log_d n + nd \log_d n)$$

(a) Show that, with the right value of $d$, the complete sequence of operations can be done in $O(n^2)$ total time.

(b) Is this asymptotic running time better than when $d = k$, for some fixed constant $k$? Why or why not?

(c) Prove that this set of operations in a d-ary heap must always take at least $\Omega(n^2)$ time.

2. CLRS 20.2-1

3. CLRS 20.4-1

4. Suppose we performed decrease-key in a Fibonacci heap using sift-up (as in a d-ary heap) instead of making cuts. For a Fibonacci heap with a tree of depth $d$, describe a worst-case sequence of $j$ decrease-key operations and show that its complexity is $\Theta(jd)$. (From the previous question, we know that in the worst-case $d = n$ for an $n$-node heap.)

5. (Grads only) CLRS 20.4-2