1. **Digital filtering.** Explain and comment on the numbers used in the following description of digital filtering of a sound system.

Suppose that we want to do a real-time Fourier transform of one channel of CD-quality sound. That’s 44k samples per second. Suppose also that we have a 1k buffer that is being re-filled with data 44 times per second. To generate a 1000-point Fourier transform we would have to perform 2 million floating-point operations (1M multiplications and 1M additions). To keep up with incoming buffers, we would need at least 88M flops (floating-point operations per second). Now, if you are lucky enough to have a 100 Mflop machine, that might be fine, but consider that you’d be left with very little processing power to spare.

Using the FFT, on the other hand, we would perform on the order of $2 \times 1000 \times \log_2(1000)$ operations per buffer, which is more like 20,000. Which requires 880k flops – less than 1 Mflop! A hundred-fold speedup.

2. **Integer multiplication.** Verify that $13 \times 11 = 143$ by performing (binary) integer multiplication using convolution. Show your computations following the method below (remember that $\omega_{1,8} = \sqrt{2}/2 + i\sqrt{2}/2$).

Given two $n$ bit integers $a = a_{n-1}a_1a_0$ and $b = b_{n-1}b_1b_0$, compute their product $c = a \times b$ through the following steps:

- Form two polynomials $A(x)$ and $B(x)$.
- Note: $a = A(2), b = B(2)$.
- Compute $C(x) = A(x)(B(x))$.
- Evaluate $C(2) = a \times b$.

Running time: $O(n \log n)$ complex arithmetic steps.

Theory. [Schönhage-Strassen 1971] $O(n \log n \log \log n)$ bit operations.

3. **Pesky probes.** Assuming that you have not succeeded in solving problem 5.7, lay out the 13 x 13 grid graph with values 1 through 169 snaking from the node (3,3) in the direction of increasing column indices (this will place value 141 in the node (13,13)) and continuing in the opposite direction from (3,3), so the first three rows will have values:

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169 168 167 166 165 164 163 162 161 160 159 158 157
144 145 146 147 148 149 150 151 152 153 154 155 156
143 142 001 002 003 004 005 006 007 008 009 010 011
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What will be the probes of a recursive algorithm which: finds the minimum value among the nodes of the border of the grid, its middle (in this case, the 7th) row and column, and the borders of the remaining four 5 x 5 grids, and descends accordingly?