Assignment 1

due Wednesday, April 14, 2010

1. Write in the \( \mathcal{S} \) language a routine making the assignment \( Y \leftarrow X_1 \) in a non-destructive manner.

2. Let \( f(0) = 0, \ f(1) = 1, \ f(2) = 2^2, \ f(3) = 3^3 = 3^{27}, \) etc. In general, \( f(n) \) is an exponential stack of \( n \)'s of height \( n \). Show that \( f \) is primitive recursive.

3. Show carefully that \( p_n \), the \( n^{th} \) prime number, is primitive recursive.

4. Let \( h(x) \) be the integer \( n \) such that

\[
n \leq x \sqrt{2} < n + 1.
\]

Show that \( h \) is primitive recursive.

5. (Strong recursion) For any function \( f \) we write \( \vec{f}(0) = 1 \) and

\[
\vec{f}(n + 1) = [f(0), f(1), \ldots, f(n)].
\]

Now let \( f(n) = g(\vec{f}(n)) \). Show that if \( g \) is primitive recursive, then \( f \) is primitive recursive.