CSE 410/510pm: Probabilistic Methods in AI

Homework #7
Due during office hours on Friday, December 3rd, 2010

Guidelines: You can brainstorm with others, but please solve the problems and write up the answers by yourself. You may use textbooks (Koller & Friedman, Russell & Norvig, etc.), your notes, and lecture slides from Winter quarter. Please do NOT use any other resources (e.g., online problem solutions) without asking.

Please show enough of your work to make your approach clear.

1. [40 pts] Consider the Rain/Sprinkler network from slide 4 of the following slides (taken from Winter quarter, based on slides by Russell and Norvig): http://www.cs.uoregon.edu/classes/10W/cis410pm/mcmc.pdf. In this problem, you will manually construct a Gibbs sampler that samples from the conditional distribution $P(C, R | S = \text{true}, W = \text{true})$.

(a) Write down the 4x4 transition matrix for the Gibbs sampler where the variable you sample is chosen randomly in each step (with 50% probability, you resample $C$, otherwise you resample $R$).

Please give the actual numbers for all transitions, along with a couple of sample calculations to show how these numbers are derived. You do not need to show your full work for every entry. (Hints: Each entry should be a probability. Some entries will be zero. The matrix will not be symmetric. See slide 15.)

(b) Compute the exact conditional probabilities for $P(C = \text{true}, R = \text{false} | S = \text{true}, W = \text{true})$ and $P(C = \text{false}, R = \text{false} | S = \text{true}, W = \text{true})$ and show that the transitions between these states satisfy the detailed balance equation.

2. [30 pts] Consider a Markov chain with two states, $x^1$ and $x^2$, and the following transition probabilities:

$$
T(x^1 \rightarrow x^1) = 1.0 - \epsilon \\
T(x^1 \rightarrow x^2) = \epsilon \\
T(x^2 \rightarrow x^1) = \epsilon \\
T(x^2 \rightarrow x^2) = 1.0 - \epsilon
$$

(a) Show that this Markov chain is regular, and that its stationary distribution is $\pi(x^1) = 0.5$, $\pi(x^2) = 0.5$.

(b) Given an initial state of $P(0)(x^1) = 1$, $P(0)(x^2) = 0$, prove that $P(t)(x^2) < t\epsilon$, where $P(t)$ is the distribution after $t$ steps.

(c) What does this say about the minimum number of samples needed to compute a reliable estimate from this Markov chain when $\epsilon$ is very small?