Homework #6
Due in class on Wednesday, November 24, 2010

Guidelines: You can brainstorm with others, but please solve the problems and write up the answers by yourself. You may use textbooks (Koller & Friedman, Russell & Norvig, etc.), your notes, and lecture slides from Winter quarter. Please do NOT use any other resources (e.g., online problem solutions) without asking.

Please show enough of your work to make your approach clear.

1. [20 pts] K&F 9.3. (Types of queries include probability of evidence $P(E = e)$ and conditional probabilities $P(Y|E = e)$.)

2. [20 pts] Consider the following factor graph and tables. Assuming that the initial messages from variables to factors are uniform (e.g., $<0.5,0.5>$), how many iterations will loopy BP take to converge? What are the resulting marginals? Are they correct? Explain your reasoning and show your work. (HINT: You don’t need to manually compute every message separately.)
3. [10 pts, Grads only] Consider a Markov network consisting of a single loop of pairwise potentials:

$$P(X_1, \ldots, X_n) = \frac{1}{Z} \phi_1(X_1, X_2) \phi_2(X_2, X_3) \ldots \phi_{n-1}(X_{n-1}, X_n) \phi_n(X_n, X_1)$$

(a) Show that eliminating any variable $X_i$ will yield an intermediate factor with a scope of three variables.

(b) Use your result from part a to prove that eliminating all variables from any network with cycles is $\Omega(\lvert Val(X_i) \rvert^3)$, where $\lvert Val(X_i) \rvert$ is the minimum number of values of any variable $X_i \in X$. (Recall that Big-$\Omega$ is analogous to Big-$O$, but a lower bound instead of an upper bound.)