Assignment 5

due Wednesday, February 27, 2008

1. Exercise 13.1-6, p 277. [6 points]

2. Give a binary tree which can be colored as a red-black tree but which is not an AVL tree. [4 points]

3. Do (i) exercise 13.3-2, p 287, (ii) then insert 35, 37, 22, (iii) and finally delete 19 and 31. Show the tree after each phase (and more if you wish). [8 points]

4. Let T be a tree storing 500,000 items. What is the worst case height of T in the following cases?
   (a) T is an AVL tree
   (b) T is a (2,4) tree
   (c) T is a red-black tree
   (d) T is a binary search tree
   [8 points]

5. Exercise 18.2-1, p 447, but change the minimum degree from 2 (which would be a (2,4)-tree) to 3. [6 points]

6. Let T and U be two red-black trees storing n and m items, respectively, such that any item in T has a key less than the keys of all items in U. Describe an \( O(\lg n + \lg m) \) method for joining the trees into a single tree that stores all the items in T and U. The original T and U may be destroyed in the process. [8 points]

7. Show how to use a heap to find the \( k^{th} \) largest of a set of \( n \) elements in \( O(n + k \log n) \) time [6 points]

Total: 46 points

Notes:

- \( Q1 \): As usual, internal node means actual node and external node means a null.
- \( Q2 \): Of course, here we are just considering the shape of the tree. You are to show a tree which could be colored as a RB tree but is too out of balance to be an AVL tree.
- \( Q6 \): Try to adapt the solution of the same (2,4)-tree problem.
- \( Q7 \): Use the fact that heap-build is \( O(n) \) and heap-delete is \( O(\log n) \).