Assignment 2

due 2007 February 1, Thursday

1. exercise 16-1, parts a and c, p 402
2. exercise 23.2-8, p 574

3. Suppose that we have a weighted graph represented by its weight matrix \( W = (w_{i,j}) \), maybe representing some model of the internet, where \( w_{i,j} \) represents the time to send a message directly from point \( i \) to point \( j \). In addition, we have vertex costs: \( c_v \) is the time it takes a message to be received and forwarded by node \( v \). Given a start node \( s \), we want to find the minimum time to send a message from node \( s \) to all other nodes.

Modify Dijkstra’s algorithm to do this. Note that at the initialization stage, you will have \( d[s] = c_s \) rather than 0.

4. Suppose that you lose your job and have to move to Mars, where it just so happens that all computers use base 3. Your new employer has a data compression problem: there is an alphabet of size \( n \), and the each of the characters occur with probability \( p_1, p_2, \ldots, p_n \). Your job is to figure out how to encode the alphabet using the values 0, 1, or 2 in a way that minimizes encoding length. Also, the encodings must be prefix-free. If you can, prove to your employer that your method is correct.

5. optional Consider a graph \( G \) with positive and distinct edge weights, and a specified vertex \( s \). Consider a MST for \( G \) as well as the tree of shortest paths from \( s \) to all other nodes (call it SPT).

(a) Are the MST and SPT always the same?
(b) Is it possible for the MST and SPT to share no edges?

Explain your answers.

6. extra extra extra credit exercise 16.4-3, p 398

Notes:

- Q3 could also model a flights and airport situation, where \( w_{i,j} \) is the time to fly from city \( i \) to city \( j \), and \( c_v \) is the time it takes to get through city \( v \)'s airport.
- That is, modify Huffman’s method to work in ternary rather than just binary.
- What extra\(^3\) means is not that you get so much extra credit but that the problem is extra hard.