Assignment 3

due Wednesday, February 7, 2007

1. Draw the binary tree whose inorder traversal is eproiuscmgdOtO and whose postorder traversal is erpisuoOgdtmc. Give the level order traversal of that tree. [5 points]

2. The balance factor of an internal node v of a binary tree is the difference between the heights of the left and right subtrees of v. Write a recursive routine which will print the balance factors of all nodes in a binary tree. What is the running time of this routine? [6 points]

3. Consider an ordered tree T and a binary tree T' representing it, using the first-child next-sibling representation (section 10.4). An inorder traversal of T' is equivalent to what kind of traversal of T? [4 points]

4. In class we defined the internal path length I and the external path length E, both measures of a binary tree. If that tree has n (internal) nodes, show that E = I + 2n. (This is exercise B.5-5, p 1091.) [8 points]

5. Consider the tree of Figure 12.2 on p 257. How many different permutations of the values it contains, when inserted in that order, will yield this particular tree? [8 points]

6. How many permutations of 1, 2, . . ., n yield a skew tree? (Since any one skew tree is generated by just one permutation, this question is asking for the number of skew trees of n nodes.) [5 points]

7. (Search path splitting a BST) Exercise 12.2-4, p 260. [4 points]

Total: 40 points

Notes:

- (Q1) The lower case and upper case of the letter ‘o’ are used so they can be distinguished.
- (Q2) Consider the following three formulas:
  - $\text{height}(\text{null}) = -1$
  - $\text{height}(p) = 1 + \max\{\text{height}(p.\text{left}), \text{height}(p.\text{right})\}$
  - $\text{balFac}(p) = \text{height}(p.\text{left}) - \text{height}(p.\text{right})$

  These suggest that you may want to compute the height and the balance factor at the same time. You may simply print out the balance factors, in any order.
• (Q3) To get $T'$, imagine the first-child as a left pointer and the next-sibling as a right pointer.

• (Q4) We had $I = \sum_{v \in V} d(v)$, where $V$ is the set of nodes and $d(v)$ is the depth of a node. $E$ is defined similarly, over all external nodes. You will want to use induction.

• (Q5) Consider a tree where
  
  – the left subtree contains $n$ nodes and is generated by $r$ permutations
  – the right subtree contains $m$ nodes and is generated by $s$ permutations

Then the whole tree contains $n + m + 1$ nodes and is generated by $r \cdot s \cdot \binom{n + m}{n}$ permutations.