Solutions to Assignment 3

1. Propositions:
R1: Mythical ⇒ Immortal
R2: ¬Mythical ⇒ ¬Immortal ∧ Mammal
R3: Immortal ∨ Mammal ⇒ Horned
R4: Horned ⇒ Magical

Translations:
R5: ¬Mythical ∨ Immortal [from R1]
R6: Mythical ∨ (¬Immortal ∧ Mammal) [from R2]
R7: (Mythical ∨ ¬Immortal) ∧ (Mythical ∨ Mammal) [from R6]
R8: ¬(Immortal ∨ Mammal) ∨ Horned [from R3]
R9: (¬Immortal ∧ ¬Mammal) ∨ Horned [from R8]
R10: (¬Immortal ∨ Horned) ∧ (¬Mammal ∨ Horned) [from R9]
R11: ¬Horned ∨ Magical [from R4]

So the clauses we have are:
C1: ¬Mythical ∨ Immortal
C2: Mythical ∨ ¬Immortal
C3: Mythical ∨ Mammal
C4: ¬Immortal ∨ Horned
C5: ¬Mammal ∨ Horned
C6: ¬Horned ∨ Magical

Combining these, we also get:
C7: Immortal ∨ Mammal [C1 and C3]
C8: ¬Mythical ∨ Horned [C1 and C4]
C9: Mythical ∨ Horned [C3 and C5]
C10: ¬Immortal ∨ Magical [C4 and C6]
C11: ¬Mammal ∨ Magical [C5 and C6]
C12: Mammal ∨ Horned [C7 and C4]
C13: Immortal ∨ Horned [C7 and C5]
C14: Mammal ∨ Magical [C7 and C10]
C15: Immortal ∨ Magical [C7 and C11]
C16: Horned [C11 and C12]
C17: Magical [C11 and C14]

Assume ¬Horned. We get a contradiction. [assumption and C16]
Therefore Horned is true.

Assume ¬Magical. We get a contradiction [assumption and C17]
Therefore Magical is true.

Assume ¬Mythical. We get:
Res1: ¬Immortal [assumption and C2]
Res2: Mammal [assumption and C3]
No other facts can be created, so we know nothing about it being not Mythical. Let’s
assume Mythical.
Res1: Immortal [assumption and C1]
No other facts can be created, so Mythical is also unknown. Therefore we cannot prove anything about Mythical being true or false.

2. Forward and backward chaining would both work in this situation since all the rules are in Horn form, and they are faster than resolution. Since we have a goal to work from, backward chaining is better, as forward chaining is going to produce a lot of extraneous information at each step.

3. Here is an example vocabulary and the corresponding sentences:

\[\text{Takes}(x,c,s): \text{student } x \text{ takes course } c \text{ in semester } s;\]
\[\text{Passes}(x,c,s): \text{student } x \text{ passes course } c \text{ in semester } s;\]
\[\text{Score}(x,c,s): \text{the score obtained by student } x \text{ in course } c \text{ in semester } s;\]
\[x > y: x \text{ is greater than } y;\]
\[F \text{ and } G: \text{specific French and Greek courses (one could also interpret these sentences as referring to any such course, in which case one could use a predicate } \text{Subject}(c,f) \text{ meaning that the subject of course } c \text{ is field } f;\]
\[\text{Buys}(x,y,z): x \text{ buys } y \text{ from } z \text{ (using a binary predicate with unspecified seller is all right but less felicitous);}\]
\[\text{Sells}(x,y,z): x \text{ sells } y \text{ to } z;\]
\[\text{Shaves}(x,y): \text{person } x \text{ shaves person } y;\]
\[\text{Born}(x,c): \text{person } x \text{ is born in country } c;\]
\[\text{Parent}(x,y): x \text{ is a parent of } y;\]
\[\text{Citizen}(x,c,r): x \text{ is a citizen of country } c \text{ for reason } r;\]
\[\text{Resident}(x,c): x \text{ is a resident of country } c;\]
\[\text{Fools}(x,y,t): x \text{ fools person } y \text{ at time } t;\]
\[\text{Student}(x), \text{Person}(x), \text{Man}(x), \text{Barber}(x), \text{Expensive}(x), \text{Agent}(x), \text{Insured}(x), \text{Smart}(x), \text{Politician}(x): \text{predicates satisfied by members of the corresponding categories.}\]

\begin{align*}
\text{a. } & \exists x \text{ Student}(x) \land \text{Takes}(x,F,Spring2001) \\
\text{b. } & \forall x,s \text{ Student}(x) \land \text{Takes}(x,F,s) \Rightarrow \text{Passes}(x,F,s) \\
\text{c. } & \exists x \text{ Student}(x) \land \text{Takes}(x,G,Spring2001) \land \forall y y \neq x \Rightarrow \neg \text{Takes}(y,G,Spring2001) \\
\text{d. } & \forall s \exists x \forall y \text{ Score}(x,G,s) > \text{Score}(y,F,s) \\
\text{e. } & \forall x \text{ Person}(x) \land (\exists y,z \text{ Policy}(y) \land \text{Buys}(x,y,z)) \Rightarrow \text{Smart}(x) \\
\text{f. } & \forall x,y,z \text{ Person}(x) \land \text{Policy}(y) \land \text{Expensive}(y) \Rightarrow \neg \text{Buys}(x,y,z) \\
\text{g. } & \exists x \text{ Agent}(x) \land \forall y,z \text{ Policy}(y) \land \text{Sells}(x,y,z) \Rightarrow (\text{Person}(z) \land \neg \text{Insured}(z)) \\
\text{h. } & \exists x \text{ Barber}(x) \land \forall y \text{ Man}(y) \land \neg \text{Shaves}(y,x) \Rightarrow \text{Shaves}(x,z) \\
\text{i. } & \forall x \text{ Person}(x) \land \text{Born}(x,UK) \land (\forall y \text{ Parent}(y,x) \Rightarrow ((\exists r \text{ Citizen}(y,UK,r)) \lor \text{Resident}(y,UK))) \Rightarrow \text{Citizen}(x,UK,Birth) \\
\text{j. } & \forall x \text{ Person}(x) \land \neg \text{Born}(x,UK) \land (\exists y \text{ Parent}(y,x) \land \text{Citizen}(y,UK,Birth)) \Rightarrow \text{Citizen}(x,UK,Descent) \\
\text{k. } & \forall x \text{ Politician}(x) \Rightarrow (\exists y \forall t \text{ Person}(y) \land \text{Fools}(x,y,t)) \land (\exists t \forall y \text{ Person}(y) \Rightarrow \text{Fools}(x,y,t)) \\
& \land \neg (\forall t \forall y \text{ Person}(y) \Rightarrow \text{Fools}(x,y,t))
\end{align*}
4. Here is a possible translation of the statements into first order logic. In general $x$ refers to a person and $y$ refers to a class.

S1: $\forall x, y \ Teaches(x,y) \land Type(y,CS) \land GoodReview(y) \Rightarrow Raise(x)$

S2: $\exists v \ Teaches(Dou,v) \land Type(v, AI)$

This sentence becomes two sentences by Existential Instantiation and introducing the constant $Class1$.

S3: $Teaches(Dou, Class1)$

S4: $Type(Class1, AI)$

S5: $\forall y \ Type(y, AI) \Rightarrow Type(y, CS)$

S6: $\forall y \ Type(y, AI) \Rightarrow GoodReview(y)$

This last statement assumes that the only AI class taught is by Professor Dou.

Forward chaining:
Substitute $y$ in S4 for $Class1$, so S4 yields: $Type(Class1, CS)$
Substitute $y$ in S5 for $Class1$, so S4 yields: $GoodReview(Class1)$
Using $\{x/Dou, y/Class1\}$, we have $Teaches(Dou, Class1) \land Type(Class1, CS) \land GoodReview(Class1) \Rightarrow Raise(Dou)$
Therefore Professor Dou does receive a raise.

Resolution:
Assume $\neg Raise(Dou)$, and convert other statements to CNF

$\neg (Teaches(x,y) \land Type(y,CS) \land GoodReview(y)) \lor Raise(x)$ [remove implication from S1]

C1: $\neg Teaches(x,y) \lor \neg Type(y,CS) \lor \neg GoodReview(y) \lor Raise(x)$ [move negation inwards]

Existential Instantiation and the introduction of constant $Class1$ yields:

C2: $Teaches(Dou, Class1)$

C3: $Type(Class1, AI)$

C4: $\neg Type(y, AI) \lor Type(y, CS)$ [Implication elimination]

C5: $\neg Type(y, AI) \lor GoodReview(y)$ [Implication elimination]

Start applying resolution.

C6: $\neg Teaches(Dou,y) \lor \neg Type(y, CS) \lor \neg GoodReview(y)$ $\{x/Dou\}$ Assumption and C1

C7: $\neg Type(Class1, CS) \lor \neg GoodReview(Class1)$ $\{x/Dou, y/Class1\}$ C2 and C6

C8: $\neg Type(Class1, AI) \lor \neg GoodReview(Class1)$ $\{x/Dou, y/Class1\}$ C4 and C7

C9: $\neg Type(Class1, AI)$ $\{x/Dou, y/Class1\}$ C5 and C8

C10: Contradiction! $\{x/Dou, y/Class1\}$ C3 and C9
Therefore $Raise(Dou)$ is true for the substitution $\{x/Dou, y/Class1\}$. 