Embarrassingly Parallel Applications

- Some history:
  - parallel processing was often viewed with suspicion
  - OK for “toy problems” but cost-effective for real work?
- Hurdles to overcome:
  - new algorithms
  - new languages (or so we thought...)
  - new machines (hypercubes, Sequent, dataflow, ...)
- The term “embarrassingly parallel” was coined to refer to problems that were “too easy” too solve
  - Geoffrey Fox (physicist at Caltech/Syracuse) [W&A]
  - Cleve Moler (mathematician, Intel/Mathworks)

Characteristics

- What makes an application “embarrassingly parallel”?
- Easy decomposition into subproblems
  - not necessarily recursive (self-similar) but that’s often the case
- Independent subproblems
  - no information passed between processes working on the subproblems
- Test: can you solve the subproblems in any order?
  - necessary but not sufficient...

Possibilities

- Static vs dynamic decomposition
- Mandelbrot project

Gordon Bell Prize

- Gordon Bell (DEC / CMU / NSF) established a prize in 1986
  - offered two $1K awards per year for 10 years
  - from the announcement of the 1988 competition:
    Application: The problems run for this test must be complete applications, no computational kernels allowed. They must contain all input, data transfers from host to parallel processors, and all output. The problems chosen should be the kind of job that a working scientist or engineer would submit as a batch job to a large supercomputer. In addition, I am arbitrarily disqualifying all problems that Cleve Moler calls "embarrassingly parallel". These include signal processing with multiple independent data streams, the computation of the Mandelbrot set, etc.
  - Prize (now $5k) still awarded each year at Supercomputing
### Performance

\[ TP = TS/n \]
\[ TP > TS/n \]

Ideal case: perfect speedup

“Master” process does decomposition
takes one piece of the work itself
collects, saves results

### Examples (W&A Ch 3)

- **Image processing**
  - a polygon can be represented by vectors \( x \) and \( y \) (coordinates of the vertices)
  - transformations involve independent operations on vector elements

  \[
  x' = x + \delta x \\
  y' = y + \delta y \\
  x' = x S_x \\
  y' = y S_y \\
  x' = x \cos \theta + y \sin \theta \\
  y' = -x \sin \theta + y \cos \theta
  \]

### Examples (cont’d)

- **Monte Carlo integration**
  - for complicated (especially multi-dimensional) functions it may be easy to compute \( f(x,y,...) \) but hard to integrate
  - MC methods generate millions of random points \((x,y,...)\)
  - the integral is \((\text{#points below the curve}) / \text{(points)}\)

  \[
  x^2 + y^2 = r^2 \\
  A = \pi r^2 \\
  \frac{\pi}{4} = \int_0^1 \sqrt{1 - x^2} \, dx
  \]
Set Definition

- The Mandelbrot set is a set of points in the imaginary plane
  \[ c = x + yi \]
- For any point \( c \) consider the recurrence
  \[ z_0 = 0 + 0i \]
  \[ z_k = z_{k-1}^2 + c \]
- For some points the values increase but for others the magnitude hovers near the origin

Set Definition (cont’d)

- If the magnitude ever goes above 2.0 we know the series will start to increase from that point on
- MS = the set of points that always stay near the origin
- Image: use color map \( M \) with \( 0 \leq N \leq 255 \) colors
- The color for point \( c \) is
  - black if series starting at \( c \) is in the set
  - color \( M[j] \) if series has \( j \) elements less than 2.0

Full Set

- The full set has lots of points on the x axis (\( b = 0 \))
- The set is a fractal
  - infinitely complex boundary
  - self-similar patterns
  - all points in the set are connected

MPI Application

- Calculation of the Mandelbrot set is clearly embarrassingly parallel
  - an image with \( n \times m \) pixels evaluates \( n \times m \) recurrences
  - each series can be evaluated independently
  - no particular order is required (row by row, column by column, ...)
- Plan (basic version):
  - process 0 gets coordinates from command line
  - process 0 allocates parts of the image to a set of other processes
  - other processes send back filled-in parts of the image
  - process 0 writes complete image to file
**Static Decomposition**

- In a static decomposition the parallel pieces are determined when the program starts.
- Example:
  - divide image into N equal-size regions
  - send each process
    - (x,y) of upper left pixel
    - dx and dy
    - width, height of region

**Inefficiency of Static Decomposition**

- It is unlikely each region will take the same amount of time.
  - regions with lots of black pixels will require more iterations.
  - region 3 will be finished before region 0...

**Dynamic Allocation (Work Pool)**

- A more efficient method: make more pieces of work, and allocate them on demand.
- Example:
  - make 16 regions
  - allocate the first four regions to the four processes
  - when a process finishes give it the next region in the "pool"
  - in this figure process 3 might compute region 4...

**Dynamic Decomposition**

- The work pool strategy on the previous slide still used a static decomposition.
  - the number of regions is fixed at the beginning.
  - dynamic allocation of regions to processes leads to better efficiency.
- For many applications it is also possible to define the jobs in the pool dynamically.
  - as the application progresses it discovers more work and adds it to the pool.
  - the application terminates when the pool is empty.
Dynamic Decomposition (cont’d)

- A trick to speed up the Mandelbrot application:
  - pick four points that define a rectangle in the image
  - evaluate the recurrence at these four points
  - if all corners are black then color all points in the interior black

- Not as accurate as full method but it’s a reasonable approximation

```
Dynamic Decomposition (cont’d)
Q = [new region(x0,xn,y0,yn)]
while not Q.empty
r = Q.remove_first
if black_corners(r)
    r.fill(black)
else
    Q.insert(new region(x0,mid(x0,xn),y0,mid(y0,yn)))
    Q.insert(new region(mid(x0,xn),xn,y0,mid(y0,yn)))
    Q.insert(new region(x0,mid(x0,xn),mid(y0,yn),yn))
    Q.insert(new region(mid(x0,xn),xn,mid(y0,yn),yn))
```

Load Balancing

- Process 0 does not always have to coordinate all tasks
- Each process can have its own local queue
- New jobs added to the local queue
- When a process runs out of jobs it can ask others for work
- Jobs can “migrate” between processes
- Many dynamic load balancing strategies

Project 2

- Download project outline (Makefile, etc) from web site
- First task: sequential implementation
  - command line args: coordinates of region
  - locations of points at corners
  - size of area covered by each pixel
- Save output in file, view with R
  - read pixel values into array
  - use `image` command to draw picture using array elements as colors
  ```r
  > A = scan("mandelbrot.rda")
  > dim(A) = c(200,200)
  > image(A)
  ```
Project 2 (cont’d)

- Simple parallel version:
  - static decomposition and allocation
  - process 0 creates subprocesses, prints results
- Develop message “protocol”
  - what information does process 0 send to others?
  - what information will be returned?
  - how can this information be sent with MPI functions?
- Develop communication pattern (hint: think “hello, world”)

NOTE: compare output of this version with sequential version

% diff mandelbrot.rda mandelbrot.seq.rda

Writeup

- All programming projects should be submitted with documentation
- Some general guidelines:
  - what did you do for this project?
  - what are the main sections of the program?
  - is there anything in particular you want me to know about your project?
  - what is the output? how did you test the program?
  - how does the program perform?
  - what would you fix or add if you were to work on it some more?

- Neatness counts, and a picture is worth.....

Extras

- Implement a dynamic allocation strategy
- Implement dynamic decomposition (e.g. “black corners trick”)
- Link with a graphics library, make an interactive program
  - select region to “zoom in”
  - new image starts out all gray
  - render pieces as they are computed
- Other ideas?

A Note About Grades

- A working program with no frills and nothing special to recommend it usually earns about 80 points (out of 100)
- To earn 100, implement one or more of the extras, or do an excellent job on the writeup (performance analysis, etc), or an especially clean and elegant implementation (style matters)
- A program with one or more major flaws, or a missing or inadequate writeup, results in 60 or less

- Course grades will be based on a curve
  - overall average of 80 is usually a B
  - to earn an A plan on doing exceptional work on at least a few projects