Midterm Exam Sample Solution

1. Sorry, I cannot nicely typeset the ranges of the summations, $\Sigma$. They should be clear from the context of the original problem.

   a) $\Sigma \Sigma j^2 = \Sigma \Theta(i^3) = \Theta(n^4)$

   b) $\Sigma i^2 n = n \ast \Sigma i^2 = n \ast \Theta(n^3) = \Theta(n^4)$

   c) After one iteration, $2^n$ becomes $\sqrt{2^n} = 2^{n/2}$. After one more, $2^{n/4}$. The question becomes: how often can we divide $n$ by 2? The answer is $\log_2 n$.

2. A node in a ternary tree will have 3 pointer fields: left, middle, and right. If there are $n$ nodes, there are $3n$ pointer fields. We know that in any tree of $n$ nodes, $n-1$ are not null (“a tree of $n$ nodes has $n-1$ edges”, covered earlier in the term). Therefore

   $3n - (n-1) = 2n + 1$

   pointer fields are null.

3.

4.

5. The following algorithm passes down the tree to an external node.

   ```java
   rangeLow = -\infty
   rangeHi = \infty

   pointer p = T.root()

   while (p != null)
   {
     if (k <= p.key)
     {
       rangeHi = p.key
       p = p.left
     }
     else
     {
       rangeLow = p.key
       p = p.right
     }
   }
   ```